

# Next-to-minimal $R$ -symmetric model: Dirac gaugino, Higgs mass and invisible width

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## Abstract

We study a singlet extension of the minimal  $U(1)_R$  symmetric model, which shares nice properties of Dirac gauginos and  $R$ -symmetric Higgs sector. At the same time, a superpotential coupling of  $R$ -charged singlet to the Higgs doublets can give a substantial contribution to the Higgs boson mass. We show that the 125 GeV Higgs boson is consistent with perturbative unification, even if the SUSY scale is as low as 1 TeV and if the  $D$ -term Higgs potential is suppressed as is often the case in Dirac gauginos. The model also contains a light scalar and fermion, pseudo-moduli and pseudo-Goldstino: The former gets a mass mainly from SUSY breaking soft terms, in addition to a small explicit  $R$ -symmetry breaking for the latter. We examine how the Higgs mass and width are affected by these light degrees of freedom. Specifically we find that depending on parameters of  $R$ -charged Higgses, the pseudo-moduli lighter than a half of the SM-Higgs boson mass is still allowed by the constraints from invisible decays of the  $Z$  and Higgs bosons. We also find that such a light scalar can reduce the Higgs boson mass, at most by a few percents.

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## I. INTRODUCTION

Low-energy supersymmetry (SUSY) is a fascinating idea since it was proposed as a solution to the hierarchy problem and further supported by a sign of gauge coupling unification. Such idea is now challenged, however, since no signal for SUSY particles has been reported yet at the Large Hadron Collider (LHC). The discovery of the Standard Model (SM)-like Higgs boson at 125 GeV also indicates that SUSY would be heavy [1], if exists, since the identification of the discovered boson with the lightest Higgs boson in the Minimal SUSY SM (MSSM) requires large radiative corrections from the top squarks.

An  $R$ -symmetric realization of low-energy SUSY provides an alternative to the MSSM [2, 3]. In particular, models with Dirac gaugino have recently attracted much attention in several respects: “supersoftness” of scalar masses [4], “supersafeness” of squark production [5, 6], as well as improved properties in flavor sector [7]: The adjoint chiral multiplets, including Dirac partners of the MSSM gauginos, also contain adjoint scalars,  $\mathcal{N} = 2$  partners of the gauginos, which have a suitable coupling to cancel UV divergences of scalar masses. This can improve naturalness property of the Higgs potential.

In the minimal setup of  $U(1)_R$ -symmetric model [7], we introduce an  $R$ -partner to each gaugino and Higgsino. An  $R$ -partner of the MSSM gaugino  $\lambda_{a=3,2,1}$ , is contained in an adjoint chiral multiplet,  $\mathcal{A}_3 = O$ ,  $\mathcal{A}_2 = T$  and  $\mathcal{A}_1 = S$ , while an  $R$ -partner of the MSSM Higgs doublet,  $H_{i=u,d}$ , is denoted by  $R_{i=u,d}$ . The minimal  $R$ -symmetric model is known to have advantages of suppressing flavor and CP violations [7–10]; Phenomenological study [11] and electroweak (EW) baryogenesis [12, 13] was also discussed. See Refs. [14–16] for  $R$ -symmetric models with different  $R$ -charge assignments, and Ref. [17] for a minimal SUSY model of Dirac gaugino with and without  $R$ -symmetry.

An important step was made in Refs. [18, 19] showing that the 125 GeV Higgs mass can be accommodated if the  $SU(2) \times U(1)$  adjoints  $\mathcal{A}_{1,2}$  have sufficiently large couplings to the Higgs pairs  $R_i H_i$ : Such adjoint Yukawa couplings can also relax the suppression of  $SU(2) \times U(1)$   $D$ -term potential, which is known as a drawback of supersoft SUSY breaking models. Notice, however, that although marginally allowed [20], such large adjoint couplings are generally in tension with EW precision measurements, as in the muleless SUSY SM [21]. Perturbativity constraints should also be taken into account.

In the present paper, we consider a singlet extension of the minimal  $R$ -symmetric SUSY SM, which may be called the next-to-minimal  $R$ -symmetric model for short; It is actually a combination of Dirac gaugino and an  $R$ -symmetric Higgs sector proposed in Ref. [22]. Unlike the conventional singlet extensions of the MSSM, the present model is not intended to explain the origin of Higgsino mass terms. Instead, we would like to examine how the lightest Higgs mass of 125 GeV can be reproduced in our  $R$ -symmetric setup. Actually we show that 125 GeV Higgs mass can well be accommodated by a singlet Yukawa coupling to the MSSM Higgs pair, consistently with perturbative unification. We also examine to what extent the adjoint Yukawa couplings can affect the results. As is expected, our solution requires small  $\tan\beta$  so that the singlet coupling can give a substantial contribution to the lightest Higgs mass; it also favors small values for the adjoint Yukawa couplings. Therefore

our setup provides an alternative to the known solution in the minimal  $R$ -symmetric model.

In the present work, we assume that explicit  $R$  symmetry breaking is very small, as was discussed in Ref. [15]; Through coupling to supergravity, the theory contains a small explicit breaking of  $R$ -symmetry, characterized by the gravitino mass, of the order of a few GeV, for instance. Throughout the present paper, we will neglect such explicit  $R$ -breaking effects, including anomaly-mediated contribution to Majorana gaugino masses. Cosmological issues will be discussed elsewhere.

The most characteristic feature of the present model is the presence of light degrees of freedom, pseudo-moduli  $\phi$  and pseudo-Goldstino  $\psi$ . In Ref. [23], properties of such light degrees of freedom were studied in some details; A generic prediction there is that the light scalar  $\phi$  couples to the Higgs boson so strongly that the Higgs decays almost invisibly to its pair. We revisit this issue here, and show that there is a region of parameter space in which a light scalar mode still exists and, unlike the previous expectation, the Higgs invisible width is within the present bound. We explain why and when the coupling of the light scalars to the Higgs boson becomes weak thanks to its pseudo-moduli nature.

It is worth mentioning that the present model has a similarity with the conventional singlet extensions of the MSSM — the NMSSM [24–28], the nMSSM [29–31] or the PQ-NMSSM [32–34]: the SM-like Higgs boson mass can be enhanced at small  $\tan\beta$ , but such enhancement is limited by the triviality bound of the singlet coupling(s). There are some important differences, however. First of all, in the present setup, the SM-like Higgs boson should be the lightest mass eigenstate in the  $R$ -neutral scalars, since the approximate  $R$ -symmetry forbids the mixing to the  $R$ -charged scalars. The  $R$ -symmetry also forbids  $A$ -terms that could be used for enhancing the Higgs mass. Moreover, in the conventional models, the Higgsino mass term is forbidden by imposing some symmetries while our model contains  $R$ -invariant Higgsino mass parameters from the start. These mass terms, on a theoretical side, trigger spontaneous SUSY breaking in the Higgs sector [22]; At the same time, the mass terms can suppress (unwanted) mixings between the singlet and doublet states in the  $R$ -charged sector. We thus expect that the resulting phenomenology can be quite different.

The present paper is organized as follows. In §II, we introduce our setup for a singlet extension of the minimal  $R$ -symmetric SUSY SM and describe its characteristic features, the existence of light degrees of freedom related to a pseudo-moduli direction. We also give a brief review of aspects of Dirac gaugino in supersoft SUSY breaking, including the suppression of  $D$ -term quartic scalar potential. In § III we discuss how the 125 GeV Higgs boson can be compatible with perturbative unification. We use two-loop renormalization group equations (RGE's) but mainly consider the case in which the SUSY particle masses can be treated by a single mass threshold  $M_S$ . We calculate the lightest Higgs mass in the RG approach: At the scale  $M_S$ , we match the  $R$ -symmetric model to the minimal SM by taking the decoupling limit of the heavier Higgs states. We also take into account various effects including adjoint Yukawa interactions and a possibility of a light pseudo-moduli below the matching scale.

The constraints from invisible decay of the  $Z$  and SM-like Higgs bosons are examined in § IV. The constraint from the  $Z$  decay to a pseudo-Goldstino pair can be satisfied if the

$R$ -symmetric Higgsino mass parameters are not too small. We also examine the invisible Higgs decay to a pseudo-moduli pair, and find that there is a region of parameter space corresponding to a pseudo-moduli that is lighter than a half of the SM-like Higgs boson mass, and such a light pseudo-moduli has very suppressed coupling to the Higgs boson(s). The final section is devoted to conclusion and discussion. Necessary tools for the RG analysis and some detailed discussion about gauge coupling unification and Dirac gaugino mass threshold are given in Appendices.

## II. DIRAC GAUGINO AND NEXT-TO-MINIMAL $R$ -SYMMETRIC HIGGS SECTOR

We consider the theory invariant under  $U(1)_R$  under which all the SM fields are neutral. The  $R$ -symmetric superpotential proposed in Ref. [22] is given by  $W = W_H + W_Y$ , where<sup>1</sup>

$$W_H = R_0 (f + \lambda H_u H_d) + \mu_u R_u H_u - \mu_d R_d H_d, \quad (1)$$

$$W_Y = y_u^{ij} Q_i U_j^c H_u + y_d^{ij} Q_i D_j^c H_d + y_e^{ij} E_i^c L_j H_d. \quad (2)$$

Here  $H_{u,d}$  are the MSSM Higgs doublets,  $R_{u,d}$  are their  $R$ -partners of  $R$  charge  $+2$ , and  $\mu_{u,d}$  are  $R$ -invariant Higgsino mass parameters which we assume to be of the order of the weak scale. A dimension-two parameter  $f$  of the order of the weak scale can act as additional source of (super)symmetry breaking, as was further studied in Refs. [15, 23]. A possible origin of these mass parameters was also discussed in Ref. [22]. In addition, we introduce a gauge singlet  $R_0$  of  $R$ -charge 2, which has an nMSSM-like coupling  $\lambda$  to the MSSM Higgs doublets  $H_{u,d}$ . We will refer to this as *singlet Yukawa coupling*. It is the purpose of the present paper to discuss the implications of this singlet Yukawa coupling in the Higgs mass and decay width.

The  $U(1)_R$  symmetry forbids the usual Majorana mass terms for the MSSM gauginos. To allow Dirac gaugino mass terms we introduce chiral supermultiplets  $\mathcal{A}_a$  belonging to the adjoint representation of the SM gauge group  $SU(3)_c \times SU(2)_L \times U(1)_Y$ , where  $a = 3, 2, 1$  is a gauge index; Explicitly,  $\mathcal{A}_3 = O$  is an  $SU(3)$ -octet,  $\mathcal{A}_2 = T$  an  $SU(2)$ -triplet,  $\mathcal{A}_1 = S$  a singlet. We denote their fermion and scalar components by  $\chi_a$  and  $A_a = (\sigma_a + i\pi_a)/\sqrt{2}$ , respectively. Matter contents and  $R$ -charge assignment are summarized in the Table. I.

In the presence of these adjoint chiral multiplets, we can add the following superpotential<sup>2</sup>

$$\begin{aligned} W_A &= \sum_{a=2,1} \sum_{i=u,d} \eta_i k_a \lambda_a^i R_i \mathcal{A}_a H_i \\ &= 2\lambda_T^u R_u T H_u - 2\lambda_T^d R_d T H_d + \lambda_S^u R_u S H_u - \lambda_S^d R_d S H_d, \end{aligned} \quad (3)$$

<sup>1</sup> The  $R$ -charged fields  $R_0$ ,  $R_u$  and  $R_d$  were denoted in Ref. [22] by  $X_0$ ,  $X_1$  and  $X_2$ , respectively. Accordingly the Higgsino mass parameters  $\mu_u$  and  $\mu_d$  here correspond to  $\mu_1$  and  $-\mu_2$  there; we have flipped the sign of  $\mu_d$  term so that all the neutral components have the same sign. Note also that the parameters,  $\lambda$ ,  $\mu_{u,d}$ ,  $f$ , can be made real and positive by field redefinition of  $R_{0,u,d}$  and  $H_{u,d}$ .

<sup>2</sup> Following the existing literature, we put a normalization factor  $k_2 = 2$  for  $SU(2)$ -adjoint Yukawa terms while  $k_1 = 1$  for  $U(1)$ . We also put sign factors  $\eta_u = +1$  and  $\eta_d = -1$  so that the neutral components have plus signs.

	$H_u$	$H_d$	$R_u$	$R_d$	$R_0$	$S$	$T$	$E_{4,5}^c$	$\overline{E}_{4,5}^c$
$SU(2)_L$	<b>2</b>	<b>2</b>	<b>2</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>3</b>	<b>1</b>	<b>1</b>
$U(1)_Y$	+1/2	-1/2	-1/2	+1/2	0	0	0	+1	-1
$U(1)_R$	0	0	+2	+2	+2	0	0	$1+r$	$1-r$

TABLE I. The charge assignments of the Higgs and extra fields, other than the  $SU(3)$ -octet  $\mathcal{A}_3 = O$ . The MSSM Higgs doublets are  $R$ -neutral while their  $R$ -partners as well as the singlet  $R_0$  have  $R$ -charge +2. The adjoint chiral multiplets  $\mathcal{A}_a$  are  $R$ -neutral so that their fermionic components have  $R$ -charge -1. All the quark and lepton superfields have  $R$ -charge +1. We also introduce two pairs of vector-like “leptons”, as is discussed in §III A.

which we refer to as *adjoint Yukawa terms* (although  $\mathcal{A}_1 = S$  is actually a singlet). In Refs. [18, 19], these terms play a central role in reproducing the lightest Higgs mass of 125 GeV. In the present model, however, it is not these adjoint Yukawa terms but the singlet Yukawa term in Eq. (1) that is important for the Higgs mass.

The superpotential  $W = W_H + W_Y + W_A$  is not completely general one that is allowed by the  $R$ -symmetry. Our assumption here is that there is no superpotential term that is quadratic in adjoint chiral multiplets  $\mathcal{A}_a$ , such as  $R_0 S^2$  and  $R_0 T^2$ . We also assume that the mixing mass term of  $S$  and  $R_0$  does not arise after GUT symmetry breaking. To justify these assumptions would require a concrete embedding of the model into grand unified theories (GUT’s): Here we just note that an interesting possibility is provided by an orbifold-type model [35], which does contain light chiral adjoints after GUT symmetry breaking.

### A. Supersoft SUSY Breaking and Suppression of $D$ -term

Dirac gaugino mass terms are generated through so-called supersoft operator [4, 36]

$$\mathcal{L}_{\text{SS}} = \sum_{a=3,2,1} \int d^2\theta \frac{\sqrt{2} g_a}{\Lambda_D} \mathcal{W}_X^\alpha \mathcal{W}_\alpha^a \mathcal{A}_a + \text{H.c.} , \quad (4)$$

where  $\mathcal{W}_X$  is the gauge field strength of a hidden sector  $U(1)_X$  whose nonvanishing  $D$ -term<sup>3</sup> is a source of SUSY breaking and  $\Lambda_D$  is a cutoff scale at which the above operator is generated. A vacuum expectation value (VEV) of  $U(1)_X$   $D$ -term,  $\langle D_X \rangle$ , gives

$$\mathcal{L}_{\text{SS}} \longrightarrow - \left\{ m_{D_a} \lambda_a \chi_a + \text{H.c.} \right\} - 2m_{D_a} D_a \sigma_a , \quad (5)$$

where  $m_{D_a} = g_a \langle D_X \rangle / \Lambda_D$  is a Dirac mass of gaugino  $\lambda_a$  and adjoint fermion  $\chi_a$  (at the scale  $\Lambda_D$ ); At the same time, the second term gives rise to the mass term of the real part  $\sigma_a$  of the adjoint scalar and the trilinear scalar interactions of  $\sigma_a$  to other scalars. The latter has important consequences, supersoftness and  $D$ -term cancellation, which we review shortly.

<sup>3</sup> See Refs. [37, 38] for a mechanism of  $D$ -term dynamical SUSY, and Refs. [39, 40] for a mechanism for generating the supersoft operator through a Wess–Zumino–Witten term. Note that a constant shift in adjoint scalars is not allowed by the supersoft operator (4).

If the  $U(1)_X$   $D$ -term is the only source of SUSY breaking, a soft scalar mass in the visible sector is UV finite. Such supersoft contribution is given by [4]

$$\delta m^2(\varphi_i) = \sum_{a=3,2,1} \frac{4C_2^a(\varphi_i) g_a^2}{16\pi^2} m_{D_a}^2 \log \frac{m_{\sigma_a}^2}{m_{D_a}^2} , \quad (6)$$

where  $C_2$  is a quadratic Casimir for a scalar  $\varphi_i$  and  $m_{\sigma_a}^2$  is the mass squared of the adjoint scalar  $\sigma_a$ . Note that its pseudo-scalar partner  $\pi_a$  does not receive a mass from the supersoft operator (4). Although the  $U(1)_R$  symmetry forbids supersymmetric mass term of  $\mathcal{A}_a$ , its scalar components  $A_a = (\sigma_a + i\pi_a)/\sqrt{2}$  can get a mass from holomorphic and non-holomorphic soft mass terms [39–42]. In addition,  $SU(3) \times SU(2)$  adjoints receive finite loop corrections (6) to their masses. We denote the resultant masses of adjoint scalars by  $m_\sigma^2 = 4m_D^2 + \delta m_\sigma^2$  and  $m_\pi^2$ , respectively. We will assume that these adjoint scalars have large enough masses to prevent them from developing a nonzero VEV especially in the presence of the adjoint Yukawa terms (3).

Another characteristic feature of Dirac gaugino models is the suppression of the  $D$ -term scalar potential [4]. Here let us recall it in the absence of adjoint Yukawa terms. Then the relevant terms are

$$V_D = \sum_a \frac{1}{2} \left[ 2m_{D_a} \sigma_a - \sum_i \varphi_i^* g_a T^a \varphi_i \right]^2 + \sum_a \left[ \frac{1}{2} \delta m_{\sigma_a}^2 (\sigma_a)^2 + \frac{1}{2} m_{\pi_a}^2 (\pi_a)^2 \right] , \quad (7)$$

where  $\varphi_i$  is a generic scalar field. If the adjoint scalar  $\sigma_a$  can be regarded as heavy enough to be integrated out, the effective  $D$ -term potential is given by

$$V_D^{\text{eff}} = \sum_a \varepsilon_{D_a} \frac{g_a^2}{2} \left[ \sum_i \varphi_i^* T^a \varphi_i \right]^2 , \quad \varepsilon_{D_a} \equiv \frac{\delta m_{\sigma_a}^2}{4m_{D_a}^2 + \delta m_{\sigma_a}^2} , \quad (8)$$

where  $\varepsilon_{D_a}$  is a suppression factor between 0 and 1. Such suppression of the quartic  $D$ -terms is not welcome in low-scale supersymmetry<sup>4</sup> and can be avoided by assuming large additional contribution  $\delta m_{\sigma_a}^2$  to the adjoint scalar masses. [Of course, this can be done at the cost of losing the supersoftness [42, 43].] Alternatively, the adjoint Yukawa terms can relax the  $D$ -term suppression [18, 19], depending on the size and sign of a combination  $\lambda_{ai}\mu_i/g_a m_{D_a}$ .

Later we will consider the common value of  $\varepsilon_{D_1} = \varepsilon_{D_2} (= \varepsilon_D)$  and treat it as a free parameter.

## B. Pseudo moduli and pseudo Goldstino

The scalar potential in the  $R$ -symmetric Higgs sector takes the form

$$V = V_F + V_A + V_D^{\text{eff}} + V_{\text{soft}} . \quad (9)$$

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<sup>4</sup> See however Refs. [44, 45] for an interesting application.

Here  $V_F$  and  $V_A$  correspond, respectively, to the superpotentials (1) and (3),

$$V_F = \sum_{I=0,u,d} |F_{Ri}|^2 + \sum_{i=u,d} |F_{Hi}|^2, \quad V_A = \sum_{a=1,2} |F_{Aa}|^2,$$

which read, explicitly for the neutral components,

$$V_F = |f - \lambda H_u^0 H_d^0|^2 + |\mu_d H_d^0|^2 + |\mu_u H_u^0|^2 + |\mu_u R_u^0 - \lambda R_0 H_d^0|^2 + |\mu_d R_d^0 - \lambda R_0 H_u^0|^2, \quad (10)$$

$$V_A = \sum_{a=1,2} |\lambda_{ai} R_i^0 H_i^0|^2 = |\lambda_S^u H_u^0 R_u^0 + \lambda_S^d H_d^0 R_d^0|^2 + |\lambda_T^u H_u^0 R_u^0 + \lambda_T^d H_d^0 R_d^0|^2. \quad (11)$$

The last term  $V_{\text{soft}}$  stands for soft scalar masses,

$$V_{\text{soft}} = \sum_{\varphi_i} m_{\varphi_i}^2 |\varphi_i|^2, \quad (12)$$

which can be induced from any  $R$ -invariant mediation of SUSY breaking as in Refs. [21, 46, 47]. Notice that in Eq. (10), we have put adjoint scalar VEV's to zero by assuming large adjoint scalar masses.

As we mentioned in §I, the model contains light degrees of freedom, pseudo-moduli  $\phi$  and pseudo-Goldstino  $\psi$ . One way to see their existence is to realize that the superpotential (1) alone defines a kind of O’Raifeartaigh model and that the corresponding scalar potential  $V_F$  has a flat direction, called pseudo moduli direction in the context of spontaneous supersymmetry breaking [48]. In the present case, it lies in the space of  $R$ -charged Higgs fields  $R_{0,u,d}$  and can be parametrized by polar angles as

$$(R_0, R_u^0, R_d^0) \sim (\cos \theta, \sin \theta \cos \varphi, \sin \theta \sin \varphi), \quad (13)$$

where, with  $\langle H_u^0 \rangle = v_u = v \sin \beta$  and  $\langle H_d^0 \rangle = v_d = v \cos \beta$ ,

$$\tan \theta = \frac{\lambda v}{\mu} \equiv \lambda v \sqrt{\frac{\cos^2 \beta}{\mu_u^2} + \frac{\sin^2 \beta}{\mu_d^2}}, \quad \tan \varphi = \frac{\mu_u}{\mu_d} \tan \beta. \quad (14)$$

Eq. (13) is not a true flat direction, and is lifted by other terms in the scalar potential (9). To discuss its impact on Higgs phenomenology is another purpose of the present paper.

Let us first discuss the pseudo-Goldstino. The present model contains nine components of neutralinos: two gauginos  $\lambda_{a=1,2}^0$  and three  $R$ -charged Higgsinos  $\tilde{R}_{0,u,d}^0$  of  $R$ -charge  $+1$ , and two Higgsinos  $\tilde{H}_{u,d}^0$  and two adjoint fermions  $\chi_{a=1,2}^0$  of  $R$ -charge  $-1$ . Among them, we are interested in the lightest mass eigenstate  $\psi$ , which is called “pseudo-Goldstino”. It can get a mass only from explicit  $R$ -symmetry breaking [15, 23], which we can neglect for the present purpose and so we treat it as a massless fermion. In the absence of the adjoint Yukawa terms (3), the pseudo-Goldstino is a mixture of  $\tilde{R}_{I=0,u,d}^0$ ; in terms of the polar angles (14),

$$\psi = \tilde{R}_0 \cos \theta + (\tilde{R}_u^0 \cos \varphi + \tilde{R}_d^0 \sin \varphi) \sin \theta \equiv \sum_{I=0,u,d} U_{\psi I} \tilde{R}_I^0. \quad (15)$$



The angle  $\theta$  is the mixing angle between  $SU(2)$ -singlet  $\tilde{R}_0$  and doublets  $\tilde{R}_{u,d}^0$ . Note that the parameter  $\mu$  defined in Eq. (14) can be regarded as a representative scale of Higgsino masses.

In passing, it is interesting to note that the pseudo-Goldstino  $\psi$  can have nonzero gaugino components in some cases. First, the inclusion of explicit  $R$ -symmetry breaking can give a tiny gaugino component [15]. In addition, the adjoint Yukawa couplings  $\lambda_{ai}$  in Eq. (3) give mixing terms between the  $R$ -charged Higgsinos  $\tilde{R}_{i=u,d}$  and the adjoint fermions  $\chi_{a=1,2}$ ,

$$-\mathcal{L}_{\psi\chi} = \sum_{a=1,2} \sum_{i=u,d} \lambda_{ai} v_i \tilde{R}_i^0 \chi_a^0 = \lambda_S^u v_u \tilde{R}_u^0 \tilde{S} + \lambda_S^d v_d \tilde{R}_d^0 \tilde{S} + \lambda_T^u v_u \tilde{R}_u^0 \tilde{T}^0 + \lambda_T^d v_d \tilde{R}_d^0 \tilde{T}^0. \quad (16)$$

where  $\chi_1^0 = \tilde{S}$  and  $\chi_2^0 = \tilde{T}^0$ . Diagonalizing it gives  $\psi$  a gaugino component of  $\mathcal{O}(\lambda_{ai} v_i / m_D)$ . Although this can be neglected in our later analysis, it may be relevant for other purposes, *e.g.*, cosmological implications of the pseudo-Goldstino.

Next we turn to the pseudo-moduli  $\phi$ , the lightest mass eigenstate in the  $R$ -charged scalars  $R_{0,u,d}^0$ . Essentially it is a fluctuation along the 'flat' direction (13), but in the presence of soft terms as well as  $D$ -terms and adjoint Yukawa terms, it can deviate from that direction. That is, if we write the pseudo-moduli state as

$$\phi \equiv \sum_{I=0,u,d} U_{\phi I} R_I^0, \quad (17)$$

the mixing angles  $U_{\phi I}$  are in general different from  $U_{\psi I}$  in Eq. (15). Accordingly, important for the pseudo-moduli mass  $m_\phi^2$  are the soft scalar masses of the  $R$ -charged scalars  $R_{0,u,d}$ . Later we will consider the case in which the doublets  $R_{u,d}$  have a common soft mass  $m_R^2$ , while the singlet  $R_0$  can have a different mass  $m_{R_0}^2$ ,

$$V_{\text{soft},R} = m_{R_0}^2 |R_0|^2 + m_R^2 \sum_{i=u,d} |R_i|^2. \quad (18)$$

When  $m_{R_0}^2 = m_R^2$ , the pseudo-moduli can be made heavy,  $m_\phi^2 = m_\psi^2 + m_R^2$ , with the same mixing angles  $U_{\phi I} = U_{\psi I}$ , as in the pseudo-Goldstino state (15).

It may be plausible that the singlet  $R_0$  does not have a soft mass as is often the case in gauge mediation of SUSY breaking. In this case,  $m_{R_0}^2 = 0$ , the pseudo-moduli gets a mass  $m_\phi^2$  through the mixing to the doublets  $R_{u,d}$ , so that  $m_\phi^2$  shows some interesting behavior. For later reference, let us elucidate the  $m_{R_0}^2 = 0$  case in the isospin symmetric case, *i.e.*, by assuming  $\tan \beta = 1$  and  $U_{\phi u} = U_{\phi d}$  so that  $\phi$  lies along  $D$ -flat direction of  $R_{u,d}$ . When the soft mass  $m_R$  can be regarded as a small perturbation, the mass eigenvalue behaves like

$$m_\phi^2 \sim m_R^2 \sin^2 \theta \sim m_R^2 \frac{\lambda^2 v^2}{\lambda^2 v^2 + \mu^2}. \quad (19)$$

Conversely, when the soft mass parameter  $m_R$  as well as  $\mu$  are much greater than  $\lambda v$ , the singlet  $R_0$  dominates the pseudo-moduli, so that its mass eigenvalue behaves like

$$m_\phi^2 \sim \lambda^2 v^2 \frac{m_R^2}{m_R^2 + \mu^2}. \quad (20)$$

In these cases, the pseudo-moduli  $\phi$  can be well lighter than the SM-like Higgs boson. We will further examine the properties of the pseudo-moduli in §IV.



### III. PERTURBATIVE UNIFICATION AND LIGHTEST HIGGS MASS

In this section, we examine gauge coupling unification and triviality bound on the lightest Higgs mass in the present  $R$ -symmetric model. Since the model is a kind of singlet extension of the minimal  $R$ -symmetric model, the most important for reproducing the Higgs mass of 125 GeV is the singlet Yukawa interaction  $\lambda R_0 H_u H_d$ , which is subject to the triviality (perturbativity) constraint<sup>5</sup> as in the conventional singlet extensions of the MSSM. To discuss the triviality bound, we need a “UV completion” of Dirac gaugino model. Here we adopt a minimal model that is consistent with perturbative unification, although we do not discuss its embedding into any concrete realizations of grand unified theories.

#### A. Gauge Coupling Unification

Gauge coupling unification is not automatic in models with Dirac gaugino since we introduce adjoint chiral multiplets  $\mathcal{A}_{a=3,2}$  which contribute to gauge beta functions  $\beta_{g_a}$ . In addition,  $R$ -charged Higgs doublets  $R_{u,d}$  are introduced. As was noted by many authors [4, 7, 17], the simplest way to recover the unification is to add two pairs of  $SU(3) \times SU(2)$ -singlets with  $U(1)_Y$ -charge  $\pm 1$ ,  $(E_{4,5}^c, \bar{E}_{4,5}^c)$ , with a mass term  $M_E E_i^c \bar{E}_i^c$  ( $i = 4, 5$ ). For definiteness, we refer to these ‘bachelor’ states<sup>6</sup> as “extra leptons”. [See Table I.] The resultant extra matter content is consistent with  $SU(3)_c \times SU(3)_L \times SU(3)_R$  trinification [50, 51]: the extra matter fields,  $\mathcal{A}_a$ ,  $R_{u,d}$  as well as the ‘bachelor’  $(E_{4,5}^c, \bar{E}_{4,5}^c)$ , can be embedded into an adjoint representation of  $SU(3)^3$ , modulo the SM singlets. Accordingly one-loop coefficients of gauge beta functions,  $(16\pi^2) dg_a/dt = b_a g_a^3$  ( $a = 3, 2, 1$ ), are shifted by the same amount,  $b_a = b_a^{\text{MSSM}} + 3$ , where  $g_1 = \sqrt{5/3} g_Y$ .

Note that the extra contribution to gauge beta functions makes the UV gauge couplings stronger than those in the MSSM. This fact is important since it relaxes triviality bound for the Higgs mass, as we shall see shortly. In this respect, the present setup is the minimal one: adding more fields would make the gauge couplings stronger at UV and further relax the triviality bound. [See Ref. [52] for another nontrivial choice of extra matter content.]

At one-loop level, gauge coupling unification is preserved, as is depicted in Fig. 1, if all the extra particles beyond the SM ones have the common mass  $M_S$  around TeV range. Here the extra particles include Dirac gauginos and scalar partners, heavier Higgses and Higgsinos, and the extra leptons, in addition to other SUSY particles.

Of course, gauge coupling unification is still nontrivial when two-loop RGE’s are used and/or masses of the extra particles are not degenerate. In the present work, however, we do not intend to examine the precision of unification to its full details, *e.g.*, by taking spectrum of extra particles into account, as was done in Ref. [17] for the constrained minimal Dirac Gaugino SUSY SM (CMDGSSM); Instead, we give a few examples in which unification is achieved to the extent that is enough for estimating the bound for the Higgs mass.

<sup>5</sup> An alternative possibility is to implement the fat Higgs idea [49] in an  $R$ -symmetric setup, but we do not pursue it here.

<sup>6</sup> Phenomenological implications of these extra “leptons” would be interesting but are beyond the scope of the present paper.

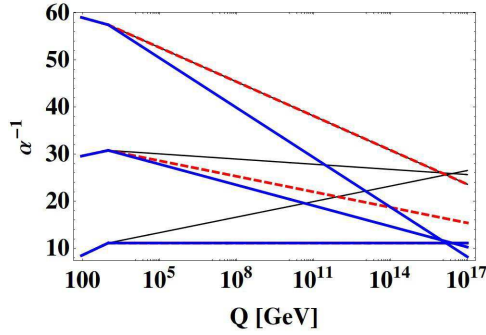


FIG. 1. One-loop running of gauge couplings in the present model (thick solid) is compared with those in the MSSM (thin solid) and the MSSM with adjoint chiral fields added (dashed). The vertical axis is  $\alpha_a^{-1} = 4\pi/g_a^2$  and horizontal axis is the renormalization scale  $Q$ .

In the following RG analysis, we use two-loop RGE's summarized in §A. In addition, we will assume that all the extra particles are approximately degenerate around the scale  $M_S = 1$  TeV or 2 TeV, except that

- (i) the extra lepton mass  $M_E$  can be different from  $M_S$ ,
- (ii) the  $SU(3)$  or  $SU(2)$  Dirac mass threshold,  $M_{D_3}$  or  $M_{D_2}$ , defined by Eq. (B1), can be different from  $M_S$ .

Unlike the heavy Majorana gluino case, a heavy Dirac gluino does not necessarily imply heavy squarks, thanks to “supersoftness” of soft scalar masses. This partially justifies our simplifying assumption as above.

Note that two-loop RG evolution of gauge couplings depends on the singlet, the adjoint and the top Yukawa couplings,  $\lambda$ ,  $\lambda_{ai}$  and  $y_t$ . Therefore the precision of unification can be discussed only after the correct value of the lightest Higgs mass is reproduced. It also depends on the suppression factor  $\varepsilon_D$  for  $D$ -term quartic since  $\varepsilon_D = 0$ , for instance, requires a larger  $\lambda$ , which implies a larger two-loop effect on gauge runnings. This being so, further details about gauge coupling unification at two-loop level are presented in §C.

## B. Triviality Bound on Singlet Yukawa Coupling

Having discussed a UV completion of the model, we now examine the triviality bound, by requiring that no coupling constant exceeds a perturbativity bound  $\sqrt{4\pi}$  up to UV cutoff  $\Lambda$ , for which we consider two cases: lower cutoff  $2.0 \times 10^{16}$  GeV and higher one  $1.0 \times 10^{17}$  GeV.

First let us focus on the case without the adjoint Yukawa couplings  $\lambda_{ai}$ . Then the quantitative behaviour can be seen from one-loop RGE's for  $\lambda$  and the top Yukawa  $y_t$ ,

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2} \left( 6y_t^2 + \lambda^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_Y^2 \right), \quad (21)$$

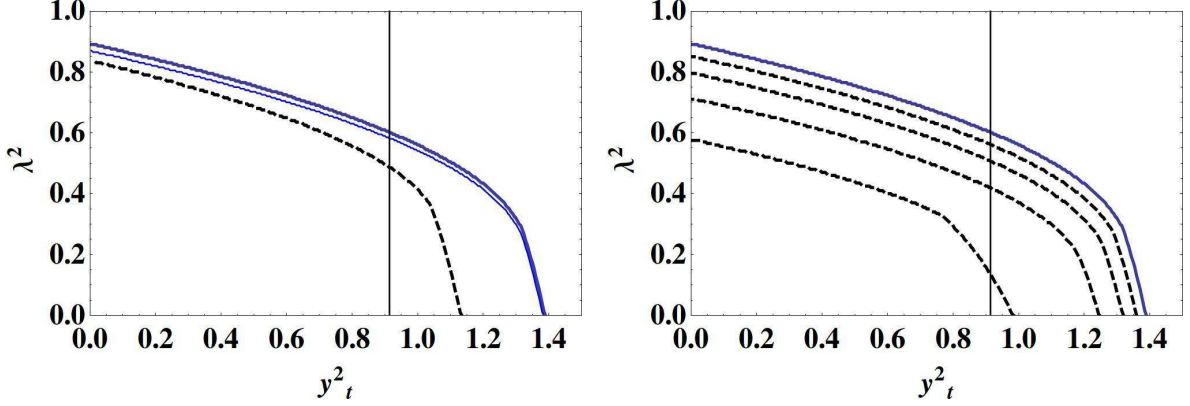


FIG. 2. Triviality bound on squared couplings  $y_t^2$  and  $\lambda^2$ . In the left, the upper solid (lower dashed) lines correspond to the present model (nMSSM), respectively. The bounds are obtained from perturbativity up to  $2.0 \times 10^{16}$  GeV, except for the thin line for  $1.0 \times 10^{17}$  GeV. In the right, the blue solid and black dashed lines correspond to  $\lambda_A = 0$  and  $\{0.2, 0.3, 0.4, 0.5\}$  from the top. In each figure, all the couplings are evaluated at  $M_S = 1$  TeV, and the vertical line corresponds to  $\tan \beta = 2$ .

$\Lambda$ [GeV]	$M_S$	$\lambda_A = 0$	0.2	0.3	0.4	0.5	nMSSM
$2 \times 10^{16}$	1 TeV	0.775	0.748	0.711	0.648	0.361	0.696
	2 TeV	0.784	0.758	0.725	0.660	0.424	0.718
$1 \times 10^{17}$	1 TeV	0.762	0.735	0.700	0.632	0.265	0.675
	2 TeV	0.771	0.745	0.707	0.644	0.339	0.595

TABLE II. Upper bound of the singlet Yukawa coupling  $\lambda(M_S)$  at  $\tan \beta = 2$ .

$$\frac{d\lambda}{dt} = \frac{\lambda}{16\pi^2} \left( 4\lambda^2 + 3y_t^2 - 3g_2^2 - g_Y^2 \right), \quad (22)$$

where  $g_Y = \sqrt{3/5}g_1$ . As we mentioned, the UV gauge coupling constants in the present model are larger than those in the MSSM or its singlet extensions. It follows that the triviality bound for  $\lambda$  as well as  $y_t$  is relaxed.

Figure 2 shows the upper bound that we obtain by using two-loop RGE's. In the left panel, the present case is compared with the nMSSM-like case whose matter content is the same as in the singlet extension of the MSSM, but without the cubic coupling  $\kappa$  of the singlet. Numerically, for  $M_S = 1$  TeV and  $\tan \beta = 2$  ( $y_t = 0.95$ ), the upper bound on  $\lambda(M_S)$  from  $\Lambda = 2.0 \times 10^{16}$  GeV is 0.775, which is improved from 0.696 in the nMSSM-like case, and becomes slightly reduced to 0.762 for  $\Lambda = 1.0 \times 10^{17}$  GeV.

We also show in the right panel of Fig. 2 how the upper bound on  $\lambda$  becomes tight when the adjoint Yukawa couplings  $\lambda_{ai}$  become large. For definiteness, we input a common value  $\lambda_A$  for  $\lambda_{ai}$  at the scale  $M_S = 1$  TeV or 2 TeV. Numerical values are shown in Table II.

### C. Triviality Bound on Higgs Mass

Now, we examine the mass of the lightest Higgs boson and calculate its upper bound. Here we adopt the RG approach, and calculate the lightest Higgs mass by matching the present  $R$ -symmetric SUSY model to a low-energy effective theory at the single scale  $M_S$ . To be more precise, starting from the input parameters summarized in §A 1, we evolve the effective theory couplings from the top mass scale to the matching scale  $M_S$ , at which we switch to the  $R$ -symmetric SUSY model. At this step, we input a value of  $\tan\beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$  to match the top Yukawa coupling. Then we evolve the SUSY couplings to UV region and require the perturbativity as before.

As for the matching scale, we take  $M_S = 1 \text{ TeV}$  or  $2 \text{ TeV}$ . Notice that this corresponds to a relatively low SUSY scale, which is still consistent with the LHC bounds thanks to “supersafeness” of Dirac gaugino scenario [5, 6, 53]. At the same time, however, it is quite nontrivial to reproduce the  $125 \text{ GeV}$  Higgs since radiative corrections from the top-stop sector are not so large;  $A$ -terms are forbidden by the  $R$ -symmetry.

As for the low-energy effective theory, we mainly consider the minimal SM model by taking the decoupling limit of heavier Higgs mass eigenstates. [We will also examine the matching to the SM coupled with a light pseudo-moduli in §III E.] Then the matching condition to the quartic Higgs potential,  $V_{\text{eff}}(H) = (\lambda_H/2) |H|^4$ , is given by

$$\lambda_H(M_S) = \frac{1}{2} \lambda^2(M_S) \sin^2 2\beta + \frac{1}{4} \varepsilon_D g_Z^2(M_S) \cos^2 2\beta, \quad (23)$$

where  $g_Z^2 = g_Y^2 + g_2^2$  and  $\varepsilon_D$  is the (common) suppression factor of  $D$ -term defined in Eq. (8).

Figure 3 summarizes our results, showing the upper bound on the lightest Higgs mass as a function of  $\tan\beta$ , for various cases. The left panel corresponds to the case without adjoint Yukawa couplings. We see that  $125 \text{ GeV}$  Higgs mass can well be reproduced at a small value of  $\tan\beta$  around 2–4. It should be emphasized that this is possible even without the  $SU(2) \times U(1)$  quartic  $D$ -terms and with the SUSY scale as low as  $1 \text{ TeV}$ .

The right panel shows to what extent the adjoint Yukawa couplings  $\lambda_{ai}$  reduce the upper bounds in the case of a common value  $\lambda_A = 0.3$ . We see that  $125 \text{ GeV}$  Higgs mass can be reproduced, but requires a larger SUSY scale ( $M_S = 2 \text{ TeV}$ ) or non-zero  $D$ -term ( $\varepsilon_D \neq 0$ ).

### D. Remarks

Some remarks are in order here.

The above result is to be compared with the nMSSM-like case, which typically requires a higher SUSY scale or sizable stop mixing [26, 31]. We also note that, unlike the existing singlet extension of the MSSM, the NMSSM or nMSSM or PQ-NMSSM [33] in which the singlet-doublet mixing can raise the mass of the SM-like Higgs boson, the approximate  $R$ -symmetry forbids such mixing between our singlet state  $R_0$  and the SM-like Higgs boson.

Another remark is that the calculated Higgs mass may be a bit underestimated especially for a low SUSY scale. Actually improved two-loop calculations show the SM-like Higgs mass

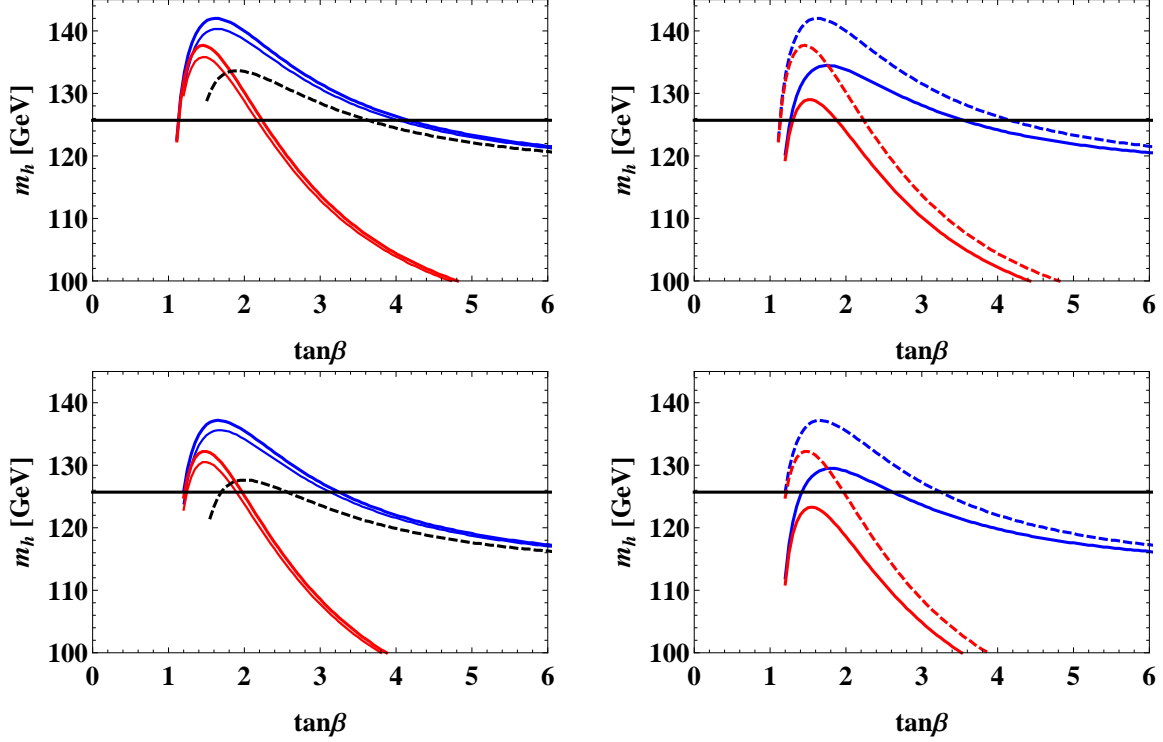


FIG. 3. Upper bounds of the lightest Higgs mass as a function of  $\tan\beta$ . The upper (lower) figures correspond to the SUSY scale  $M_S = 2\text{ TeV}$  ( $M_S = 1\text{ TeV}$ ), respectively. In each figure, upper blue (lower red) lines correspond to the unsuppressed  $D$ -term  $\varepsilon_D = 1$  (completely suppressed  $D$ -term  $\varepsilon_D = 0$ ), respectively. In the left panel, we take  $\lambda_A = 0$ , and the dashed line corresponds to the nMSSM-like case. The bounds are obtained from perturbativity up to  $\Lambda = 2.0 \times 10^{16}\text{ GeV}$ , except for the thin lines corresponding to  $\Lambda = 1.0 \times 10^{17}\text{ GeV}$ . In the right panel, we compare the  $\lambda_A = 0.3$  case (solid lines) with the  $\lambda_A = 0$  case (dashed lines).

receives a significant correction in the NMSSM [28] if the singlet is light, and also in the minimal  $R$ -symmetric model (MRSSM) [20]. Nevertheless, it is also clear that the present model gives a sufficiently large improvement.

Finally let us briefly discuss to what extent the adjoint Yukawa couplings are allowed for realizing 125 GeV Higgs mass. Figure 4 shows contours of the Higgs masses in the space of  $(\tan\beta, \lambda_A)$ , for a larger  $M_S = 2\text{ TeV}$ . As is seen from the left panel, if we input a common value of adjoint Yukawa couplings  $\lambda_{S,T}^{u,d} = \lambda_A$  at the scale  $M_S$ , its upper limit is around  $\lambda_A \sim 0.3\text{--}0.4$  depending on the  $D$ -term suppression factor  $\varepsilon_D$ . The right panel also shows that compared with the left,  $\lambda_T$  can be about 15 % larger if  $\lambda_S^{u,d} = 0$ , while  $\lambda_S$  can be about 38 % larger if  $\lambda_T^{u,d} = 0$ .

Notice that  $\lambda = 0$  corresponds to the MRSSM, where the Higgs mass can be reproduced in a large  $\tan\beta$  region. In this sense, there are two possibilities in  $R$ -symmetric models:

- (i) large  $\tan\beta$  solution: the 125 GeV Higgs can be reproduced by the large adjoint Yukawa couplings and corresponding radiative corrections.
- (ii) small  $\tan\beta$  solution: the 125 GeV Higgs can easily be reproduced by the large singlet

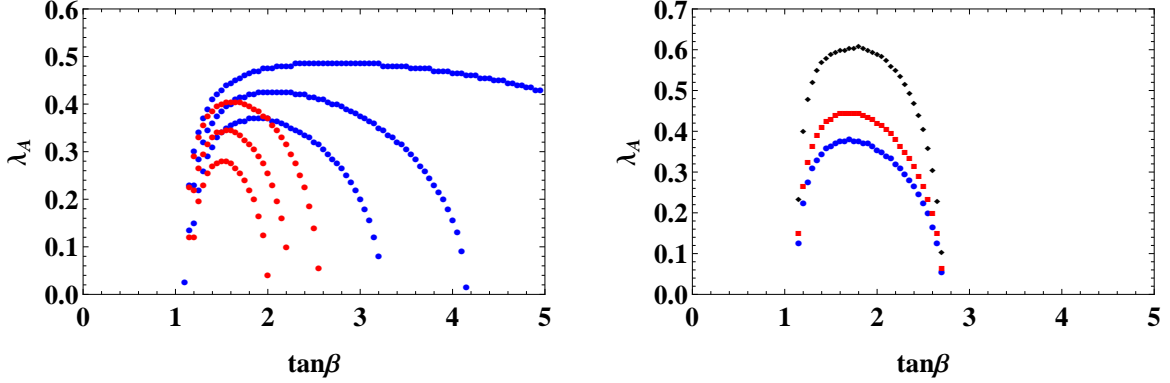


FIG. 4. The upper bounds of adjoint Yukawa couplings for  $M_S = 2\text{ TeV}$  case. In the left, the upper bounds corresponding to the Higgs mass 120, 125 and 130 GeV are shown from the top, by blue dots ( $\varepsilon_D = 1$ ) and red dots ( $\varepsilon_D = 0$ ), respectively. In the right ( $\varepsilon_D = 0.5$ ), the contours of the Higgs mass 125 GeV are shown for three cases  $(\lambda_S^{u,d}, \lambda_T^{u,d}) = (\lambda_A, 0)$ ,  $(0, \lambda_A)$  and  $(\lambda_A, \lambda_A)$ , from the top by black, red and blue dots, respectively.

Yukawa coupling (and small adjoint couplings).

The former requires nontrivial mass splitting within  $R$ -charged Higgses and  $SU(2) \times U(1)$  adjoints so that the large adjoint coupling(s) can play a similar role as the top Yukawa  $y_t$  in the MSSM. In our treatment, such effects are not incorporated since we are treating the SUSY spectrum as a single mass threshold.

### E. Matching to the SM coupled with Pseudo-Moduli

As we described in §II B, the present model contains a light degree of freedom, pseudo-moduli  $\phi$ , which is a complex scalar and can couple to the SM-like Higgs field. Therefore it can affect the mass of the SM-like Higgs. To estimate this effect, we consider the effective theory that contain the SM and the pseudo-moduli,

$$V_{\text{eff}}(H, \phi) = \frac{1}{2} \lambda_H |H|^4 + \lambda_{\phi H} |\phi|^2 |H|^2 + \lambda_{\phi} |\phi|^4. \quad (24)$$

It is not clear to us which field should be identified with the moduli field  $\phi$  in a symmetric phase of the effective theory since the singlet  $R_0$  can not mix with the doublets  $R_{u,d}$  before the EWSB. Recall from §II B, however, that the pseudo-moduli  $\phi$  is light especially when the Higgsino mass parameter  $\mu$  is large and the soft mass  $m_R$  is small. In this situation, the singlet scalar  $R_0$  dominates the pseudo-moduli mass eigenstate. So we construct the low-energy effective theory by identifying the pseudo-moduli  $\phi$  with the singlet  $R_0$ .

To be specific, let us assume  $\mu = \mathcal{O}(M_S)$  and integrate out the heavy doublet fields  $R_{u,d}$ . This leads to the tree-level matching condition (23) supplemented by

$$\lambda_{\phi H}(M_S) = \varepsilon_R \lambda^2(M_S), \quad \lambda_{\phi}(M_S) = 0, \quad (25)$$

$$\varepsilon_R \equiv \frac{m_{R_u}^2}{\mu_u^2 + m_{R_u}^2} \cos^2 \beta + \frac{m_{R_d}^2}{\mu_d^2 + m_{R_d}^2} \sin^2 \beta , \quad (26)$$

where  $m_{R_{u,d}}$  are the soft masses of  $R_{u,d}$ . Here we have introduced a factor  $\varepsilon_R$  which represents a suppression of the “portal” coupling  $\lambda_{\phi H}$  at the matching scale. This is a kind of non-decoupling effect proportional to soft SUSY breaking parameters,  $\varepsilon_R \rightarrow m_R^2 / (\mu^2 + m_R^2)$  for  $\mu_u = \mu_d$  and  $m_{R_u}^2 = m_{R_d}^2$ , which is small if  $m_R \ll \mu$ . On the other hand, it can be of  $\mathcal{O}(1)$  if  $m_R \gg \mu$ . Even in this case, Eq. (20) shows that the  $\phi$  can remain light if  $m_{R_0} = 0$ . Then the “portal” coupling  $\lambda_{\phi H}$  is quite large at  $M_S$ .

For definiteness, we examine the extreme case in which the pseudo-moduli is lighter than the SM-like Higgs boson. Then we evolve the couplings in the effective theory (24) from  $M_S$  down to the Higgs mass scale, by using two-loop RGE’s shown in §A 3. Here we show one-loop parts, which already contain the most important term:

$$\beta_{\lambda_H}^{(1)} = \beta_{\lambda_H}^{(1)} \Big|_{\text{SM}} + 2\lambda_{\phi H}^2 , \quad (27)$$

$$\beta_{\lambda_{\phi H}}^{(1)} = \lambda_{\phi H} \left( 6\lambda_H + 4\lambda_{\phi H} + 2\lambda_{\phi} + 6y_t^2 - \frac{9}{2}g_2^2 - \frac{15}{2}g_Y^2 \right) , \quad (28)$$

$$\beta_{\lambda_{\phi}}^{(1)} = 8\lambda_{\phi H}^2 + 5\lambda_{\phi}^2 , \quad (29)$$

where the SM contributions are given by

$$\beta_{\lambda_H}^{(1)} \Big|_{\text{SM}} = 12\lambda_H^2 + 12\lambda_H y_t^2 - 12y_t^4 - (3g_Y^2 + 9g_2^2) \lambda_H + \frac{3}{4}g_Y^4 + \frac{3}{2}g_Y^2 g_2^2 + \frac{9}{4}g_2^4 , \quad (30)$$

$$\beta_{y_t}^{(1)} = y_t \left( \frac{9}{2}y_t^2 - \frac{17}{12}g_Y^2 - \frac{9}{4}g_2^2 - 8g_3^2 \right) . \quad (31)$$

Eq. (27) clearly shows that the portal coupling  $\lambda_{\phi H}$  has an effect of reducing the Higgs mass.

Figure 5 shows our result. We plot the  $\varepsilon_R = 1$  case by the solid line, which is to be compared with the  $\varepsilon_R = 0$  shown by the dashed line. Numerically, if we compare these two cases by the peak values of the calculated Higgs mass, then the reduction is of 1.17% (1.57%) for  $M_S = 1$  TeV (2 TeV) for unsuppressed  $D$ -term case  $\varepsilon_D = 1$ , while it is reduced by 1.07% (1.40%) for completely suppressed  $D$ -term  $\varepsilon_D = 0$ . We see that the inclusion of the pseudo-moduli in the low-energy effective theory does reduce the Higgs mass, but such effect is only at most of a few percents. As is expected, such reduction becomes important when the matching scale becomes large.

#### IV. CONSTRAINTS FROM INVISIBLE DECAYS

In this section, we examine how the present model is constrained from invisible decays of the Higgs boson [54] as well as the  $Z$  boson. To calculate the mass and coupling of the pseudo-moduli, we use as an input value at  $M_S = 1$  TeV of the singlet Yukawa coupling  $\lambda = 0.73$  that corresponds, at  $\tan \beta = 2$ , to the lightest Higgs mass 125 GeV. No radiative correction is taken into account in this section.



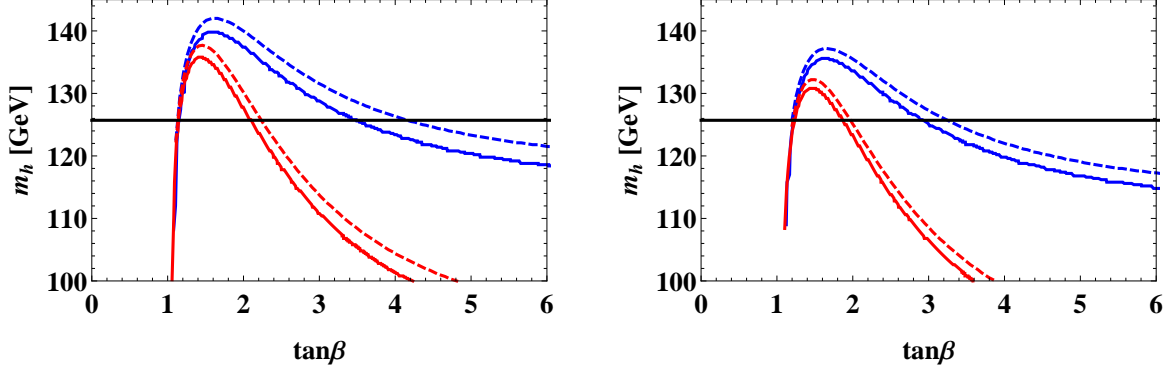


FIG. 5. Upper bounds of the lightest Higgs mass as a function of  $\tan\beta$ . In each figure, the matching to the SM with a light moduli is assumed in the solid lines with the unsuppressed matching condition  $\varepsilon_R = 1$ , while the dashed lines correspond to the matching to the SM ( $\varepsilon_R = 0$ ), which are the same as the solid ones in the left panel of Fig. 3. The right (left) panel corresponds to the SUSY scale  $M_S = 2 \text{ TeV}$  ( $M_S = 1 \text{ TeV}$ ), and as before, the blue and red lines correspond to  $\varepsilon_D = 1$  case and  $\varepsilon_D = 0$  case, respectively.

### A. Mass of Pseudo-Moduli

The present singlet extension of the minimal  $R$ -symmetric model contains a light scalar corresponding to a fluctuation along the “pseudo-moduli” direction. As was described in §II B, the pseudo-moduli  $\phi$  can receive a mass  $m_\phi^2$  from various sources in the scalar potential (9): the  $D$ -term, adjoint Yukawa terms, as well as soft scalar masses of  $R$ -charged Higgses:

$$m_\phi^2 = m_{\phi,F}^2 + m_{\phi,D}^2 + m_{\phi,A}^2 + m_{\phi,\text{soft}}^2. \quad (32)$$

For soft term, we assume the form (18),

$$V_{\text{soft},R} = m_{R_0}^2 |R_0|^2 + m_R^2 \sum_{i=u,d} |R_i|^2, \quad (33)$$

where the soft mass  $m_{R_0}$  of the singlet  $R_0$  can be different from the common soft mass  $m_R$  of the doublets  $R_{u,d}$ . When all the  $R$ -charged Higgses have a universal soft mass  $m_{R_0}^2 = m_R^2 (= m^2)$ , the pseudo-moduli has a mass equal to  $m_\phi^2 = m^2$  and its mixing angles coincide with those of the pseudo-Goldstino states (13),  $U_{\phi I} = U_{\psi I}$  ( $I = 0, u, d$ ).

We are particularly interested in the limiting case,  $m_{R_0}^2 \ll m_R^2$ . Figure 6 shows the mass eigenvalue of the pseudo-moduli state  $\phi$ , in three particular cases,  $m_{R_0}^2 = 0$ ,  $m_{R_0}^2 = 0.2 m_R^2$  and  $m_{R_0}^2 = 0.5 m_R^2$ . From the figure, we see the following:

- For a fixed value of  $\mu$ , the pseudo-moduli mass  $m_\phi$  increases by increasing the soft mass  $m_R$  of  $R_{u,d}$ , and approaches the singlet mass  $m_{R_0}$  in the limit of large  $m_R$ . In particular, if  $m_{R_0} = 0$ , it approaches the limiting value  $\lambda v$ , which is 126 GeV for  $\lambda = 0.73$ .
- For a fixed value of  $m_R$ , its mass  $m_\phi$  decreases by increasing  $\mu$ . This is because the singlet component dominates the pseudo-moduli state when  $\mu$  is large compared to  $\lambda v$ .

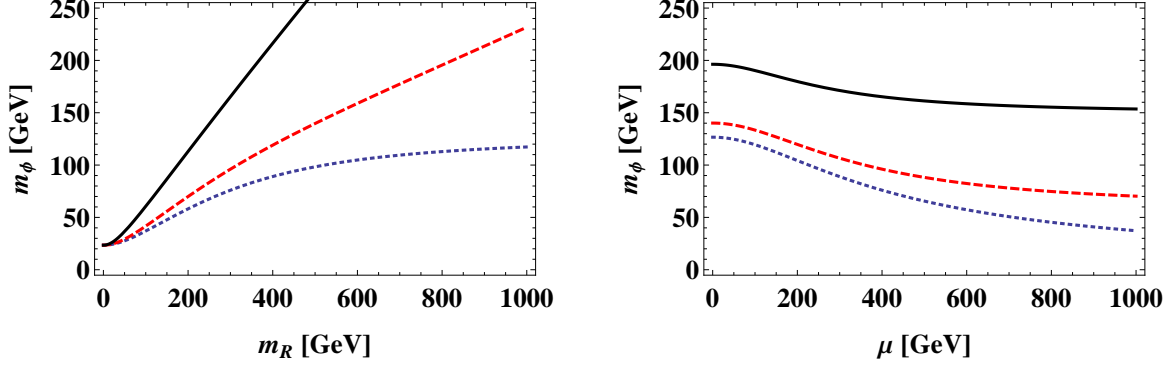


FIG. 6. The pseudo-moduli mass  $m_\phi$  as a function of the averaged  $\mu$  parameter (left) and the common doublet soft mass  $m_R$  (right). We take  $\lambda = 0.73$  and  $\tan\beta = 2$ ;  $\mu = 400$  GeV on the left, while  $m_R = 300$  GeV on the right. For the singlet soft mass  $m_{R_0}$ , three cases  $m_{R_0}/m_R = 0, 0.2$ , and  $0.5$  are shown by blue dotted, red dashed and solid lines, respectively.

These behavior can be understood from our estimate (19)–(20).

## B. Interactions to the SM Higgs

Next we calculate the trilinear coupling  $g_{h\phi\phi}$  of the lightest Higgs boson  $h$  to the pseudo-moduli pair, which also takes the form

$$g_{h\phi\phi} = g_{h\phi\phi}^{(F)} + g_{h\phi\phi}^{(D)} + g_{h\phi\phi}^{(A)}. \quad (34)$$

Let us take a close look at the first term  $g_{h\phi\phi}^{(F)}$  corresponding to the superpotential (1). Substituting the usual expansion

$$\begin{pmatrix} H_u \\ H_d \end{pmatrix} = v \begin{pmatrix} \sin\beta \\ \cos\beta \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ H' \end{pmatrix}$$

into the corresponding term  $V_F$  in the scalar potential (9), we obtain terms quadratic in the pseudo-moduli field  $\phi$  as

$$V_{\phi,F} = m_{\phi,F}^2 |\phi|^2 + g_{h\phi\phi}^{(F)} |\phi|^2 h + \frac{1}{2} g_{hh\phi\phi}^{(F)} |\phi|^2 h^2. \quad (35)$$

Here the contributions to the mass and the trilinear coupling are given by

$$m_{\phi,F}^2 = |\delta_u m_\phi|^2 + |\delta_d m_\phi|^2, \quad (36)$$

$$g_{h\phi\phi}^{(F)} = \frac{\lambda}{\sqrt{2}} \left[ U_{0\phi}^* \left( \delta_u m_\phi \sin\alpha - \delta_d m_\phi \cos\alpha \right) + \text{c.c.} \right], \quad (37)$$

where  $U_{I\phi} = U_{\phi I}^*$  is the component of the lowest mass eigenstate  $\phi$  in the fields  $R_{I=0,u,d}$ , and we have defined

$$\delta_u m_\phi \equiv \frac{\partial F_{H_u}}{\partial \phi} = \mu_u U_{u\phi} - \lambda v_d U_{0\phi},$$

$$\delta_d m_\phi \equiv \frac{\partial F_{H_d}}{\partial \phi} = \mu_d U_{d\phi} - \lambda v_u U_{0\phi} . \quad (38)$$

The quartic coupling  $g_{hh\phi\phi}$  can be found in a similar manner; we omit it here since it is irrelevant for later purposes.

Now, it is important to realize that the pseudo-moduli mixing angles  $U_{I\phi}$  ( $I = 0, u, d$ ) coincide with pseudo-Goldstino angles (13),  $U_{I\phi} \rightarrow U_{I\psi}$ , if we neglect the terms in the scalar potential (9) other than  $V_F$ ; The same is true if the soft masses are universal in the  $R$ -charged Higgs sector. In these situations, we have

$$\begin{aligned} \delta_u m_\phi &= -\mu \sin \theta \cos \beta \left[ \frac{U_{0\phi}}{\cos \theta} - \frac{U_{u\phi}}{\sin \theta \cos \varphi} \right] \longrightarrow 0 , \\ \delta_d m_\phi &= -\mu \sin \theta \sin \beta \left[ \frac{U_{0\phi}}{\cos \theta} - \frac{U_{d\phi}}{\sin \theta \sin \varphi} \right] \longrightarrow 0 , \end{aligned} \quad (39)$$

which reflects the fact that the pseudo-moduli  $\phi$  does not get a mass from the Higgs VEV's. It implies that  $\phi$  decouples from the Higgs boson in this limit. In other words, the trilinear interaction of a lightest Higgs and two pseudo-moduli arises from a deviation of the pseudo-moduli eigenstate from the would-be flat direction, caused by the other terms in the scalar potential (9).

The  $D$ -term contribution to the trilinear coupling is<sup>7</sup>

$$g_{h\phi\phi}^{(D)} = -\varepsilon_D \frac{g_Z^2 v}{2\sqrt{2}} \sin(\alpha + \beta) \left( |U_{u\phi}|^2 - |U_{d\phi}|^2 \right) . \quad (40)$$

This contribution is quite small, however, in most of the parameter space: When  $\mu$  or  $m_R$  is larger than  $\lambda v \sim 126$  GeV, the pseudo-moduli state is dominated by the singlet component, so that the doublet components  $U_{i\phi}$  ( $i = u, d$ ) are small. Moreover, in the decoupling limit of the heavier Higgses, we have  $\sin(\alpha + \beta) \rightarrow -\cos 2\beta$  ( $\sim 0.6$ ), giving another suppression for a small  $\tan \beta$  ( $\sim 2$ ).

The adjoint Yukawa terms (11) also contribute, after the EW symmetry breaking, to the mass matrix of  $R$ -charged Higgses and the trilinear coupling to the Higgs boson

$$g_{h\phi\phi}^{(A)} = \sum_{a=S,T} \frac{v}{\sqrt{2}} \left[ (\lambda_a^u U_{u\phi} \sin \beta + \lambda_a^d U_{d\phi} \cos \beta) (\lambda_a^u U_{u\phi} \cos \alpha - \lambda_a^d U_{d\phi} \sin \alpha)^* + \text{c.c.} \right] . \quad (41)$$

This contribution is also small unless  $\lambda_A v$  is comparable to  $\mu$  or  $m_R$ .

Figure 7 shows the trilinear coupling  $g_{h\phi\phi} = g_{h\phi\phi}^{(F)} + g_{h\phi\phi}^{(D)} + g_{h\phi\phi}^{(A)}$  of the lightest Higgs and two pseudo-moduli. We see that it is a increasing function of the soft mass  $m_R$  as is expected; it also decreases as  $\mu$  becomes large, since the singlet  $R_0$  dominates in the eigenstate  $\phi$ .

In passing, we note that In the decoupling limit of heavier Higgses, where  $\alpha \rightarrow \beta - \pi/2$ , Eqs. (35), (40)–(41) are slightly simplified as

$$g_{h\phi\phi}^{(F)} \longrightarrow \sqrt{2} \lambda^2 v |U_{0\phi}|^2 - \frac{\lambda}{\sqrt{2}} \left[ U_{0\phi}^* (\mu_u U_{u\phi} \cos \beta + \mu_d U_{d\phi} \sin \beta) + \text{c.c.} \right] , \quad (42)$$

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<sup>7</sup> This contribution can take both signs, as in the case of  $D$ -contributions to soft scalar masses.

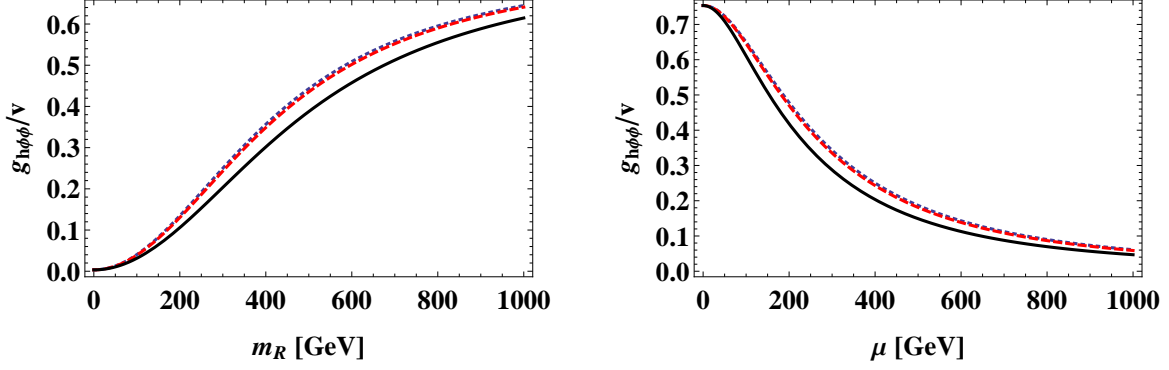


FIG. 7. The trilinear coupling  $g_{h\phi\phi}$  (divided by the Higgs VEV) as a function of  $\mu$  and soft mass  $m_R$  of  $R_{u,d}$ . The parameters are the same as in the previous figure 6.

$$g_{h\phi\phi}^{(D)} \longrightarrow \sqrt{2} \frac{g_Z^2}{4} v \cos 2\beta \left( |U_{u\phi}|^2 - |U_{d\phi}|^2 \right), \quad (43)$$

$$g_{h\phi\phi}^{(A)} \longrightarrow \sqrt{2} \sum_{a=S,T} v \left| \lambda_a^u U_{u\phi} \sin \beta + \lambda_a^d U_{d\phi} \cos \beta \right|^2. \quad (44)$$

### C. Bounds from Invisible Decays

Having calculated the mass and the interaction of the pseudo-moduli, we now examine the constraints from the invisible decays.

First let us briefly discuss the constraint from invisible decay of the  $Z$  boson. Since we are supposing that explicit  $R$  symmetry breaking is very small [15], we treat the pseudo-Goldstino  $\psi$  as massless in the following.

The  $Z$  coupling of the pseudo-Goldstinos comes from the neutral current

$$J_Z^\mu|_{\tilde{R}} = \frac{g_Z}{2} \left[ \tilde{R}_d^{0\dagger} \bar{\sigma}^\mu \tilde{R}_d^0 - \tilde{R}_u^{0\dagger} \bar{\sigma}^\mu \tilde{R}_u^0 \right] = -\frac{g_Z}{2} \psi^\dagger \bar{\sigma}^\mu \psi \sin^2 \theta \cos 2\varphi + \dots, \quad (45)$$

with the mixing angles  $\theta$  and  $\varphi$  defined by Eq. (14). The decay width of  $Z \rightarrow \psi\psi^\dagger$  is then

$$\Gamma_{Z \rightarrow \psi\psi} = \frac{g_Z^2 m_Z}{96\pi} (\sin^2 \theta \cos 2\varphi)^2. \quad (46)$$

Requiring that this is less than the error of the measured value [55] of the total width  $\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$ , we obtain the bound

$$|\sin^2 \theta \cos 2\varphi| \leq 0.0988. \quad (47)$$

The bound is depicted in Fig. 8. Recall that  $\tan \theta = \lambda v / \mu$  is the mixing angle between the singlet and doublet components of  $R$ -charged Higgsinos,  $\tilde{R}_0$  and  $\tilde{R}_{u,d}^0$ . For a generic value of  $\tan \varphi$ , which is the angle between  $\tilde{R}_u$  and  $\tilde{R}_d$ , a smaller value of  $\mu$  implies that the pseudo-Goldstino  $\psi$  has a larger doublet component and thus more strongly couples to the  $Z$  boson. We see that the constraint can easily be satisfied if the averaged Higgsino mass

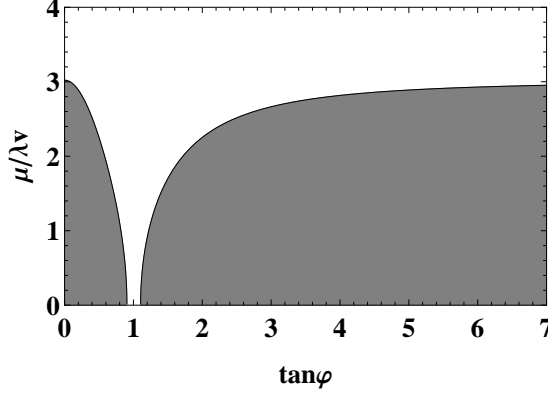


FIG. 8. The constraint from invisible decay of the  $Z$  boson. The gray region is excluded. The vertical axis is the mixing angle  $\cot \theta = \mu/\lambda v$  between the singlet/doublet components of  $R$ -charged Higgsinos,  $\tilde{R}_0$  and  $\tilde{R}_{u,d}^0$ , while the horizontal axis is the angle  $\tan \varphi$  within  $\tilde{R}_{u,d}^0$ .

parameter  $\mu$  is larger than  $\lambda v$ : For  $\lambda = 0.73$ , we get a lower bound  $\mu \gtrsim 285 \text{ GeV}$ . Then the chargino mass bounds from direct searches are also satisfied. We add that the pseudo-Goldstino becomes almost decoupled from the  $Z$  boson for  $\tan \varphi = (\mu_u/\mu_d) \tan \beta \approx 1$ , or equivalently, for  $\tan \beta \approx \mu_d/\mu_u$  as is the case if the soft masses of  $H_{u,d}$  are small.

The  $Z$  boson can decay also into a pair of pseudo-moduli  $\phi$  if the latter is lighter than  $m_Z/2$ . This corresponds to  $m_R \lesssim 150 \text{ GeV}$  for  $\mu = 400 \text{ GeV}$  and  $\mu \gtrsim 700 \text{ GeV}$  for  $m_R = 300 \text{ GeV}$ , as is seen from Fig. 6. The partial decay width for  $Z \rightarrow \phi\phi^\dagger$

$$\Gamma_{Z \rightarrow \phi\phi} = \frac{g_Z^2 m_Z}{96\pi} (|U_{u\phi}|^2 - |U_{d\phi}|^2)^2 \left(1 - \frac{4m_\phi^2}{m_Z^2}\right)^{3/2} \quad (48)$$

approaches that of  $Z \rightarrow \psi\psi^\dagger$  in the limit of small  $m_R$ , but becomes negligible if the soft mass  $m_R$  is as large as  $100 \text{ GeV}$ .

Next we discuss the invisible width of the Higgs boson. If the pseudo-moduli  $\phi$  is lighter than  $m_h/2$ , the Higgs boson can decay into a pair of  $\phi$ . The decay width for  $h \rightarrow \phi\phi^\dagger$  is given in terms of the trilinear coupling  $g_{h\phi\phi}$  and the pseudo-moduli mass  $m_\phi$  by<sup>8</sup>

$$\Gamma_{h \rightarrow \phi\phi} = \frac{g_{h\phi\phi}^2}{16\pi m_h} \sqrt{1 - \frac{4m_\phi^2}{m_h^2}}. \quad (49)$$

We require that this width should be smaller than the partial width of  $h \rightarrow b\bar{b}$  in the SM,

$$\Gamma_{h \rightarrow \phi\phi} < \Gamma_{h \rightarrow b\bar{b}}^{\text{SM}} \approx 2.34 \times 10^{-3} \text{ GeV}, \quad (50)$$

where  $\Gamma_h^{\text{SM}} = 4.07 \times 10^{-3} \text{ GeV}$  and  $\text{Br}(h \rightarrow b\bar{b}) = 57.7\%$  are the SM predictions [55].

<sup>8</sup> The decay  $h \rightarrow \psi\bar{\psi}$  is negligible since it is proportional to  $m_\psi$  (modulo a loop-suppressed contribution) and  $m_\psi$  vanishes in the  $R$ -symmetric limit.

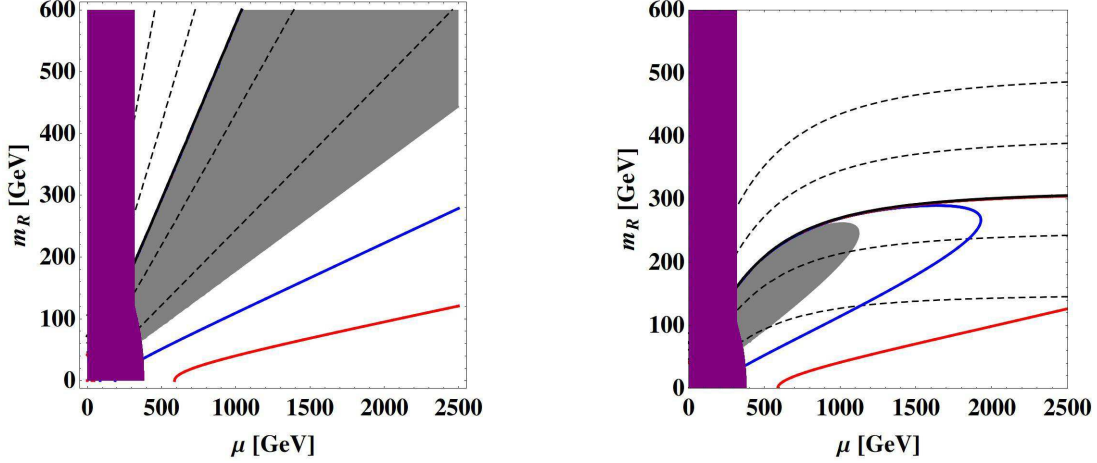


FIG. 9. The constraints from invisible  $Z$  and Higgs decays in  $(\mu, m_R)$  plane. The input parameters are  $\tan \beta = 2$ ,  $\tan \varphi = 2.5$ ,  $\varepsilon_D = 0.5$  and  $\lambda = 0.73$  at  $M_S = 1$  TeV. In the left panel, we take  $m_{R_0} = 0$  while  $m_{R_0} = 0.2 m_R$  in the right. The purple and gray regions are excluded by invisible decay of the  $Z$  and Higgs, respectively. The black solid contour corresponds to  $m_\phi = m_h/2$ , while the dashed contours correspond to  $m_\phi = \{30, 50, 80, 100\}$  GeV, respectively, from the bottom. Future reaches of the LHC ( $\text{Br} = 0.09$ ) and ILC ( $\text{Br} = 0.004$ ) are also shown by the blue and red lines, respectively.

Figure 9 shows the resulting bound on parameters  $\mu$  and  $m_R$ . The left panel corresponds to the  $m_{R_0} = 0$  case, in which an excluded region appears once the pseudo-moduli mass  $m_\phi$  (shown by the dashed lines) is smaller than  $m_h/2$ . Remarkably and unlike a generic expectation, the constraint becomes weak and disappears as the pseudo-moduli becomes lighter and lighter. We also see from the right panel that the constraint is milder for the singlet soft mass  $m_{R_0} \neq 0$ . Actually it disappears if the  $\mu$  parameter is as large as 1 TeV.

In the figures, we also show the contours of future reaches for the branching ratio of invisible Higgs decays, for which we quote 9% for the LHC with  $3000 \text{ fb}^{-1}$  at  $\sqrt{s} = 14$  TeV [56], and 0.4% for the ILC with  $1150 \text{ fb}^{-1}$  at  $\sqrt{s} = 250$  GeV [57]. We need other constraints, for instance from direct searches for charginos, to cover the whole parameter space.

The reason for this behavior is that lighter  $\phi$  implies smaller  $g_{h\phi\phi}$  coupling. As a comparison, we also show in Fig. 10 the corresponding bound in the case of  $\lambda_A = 0.4$ . In this case, the would-be pseudo-moduli direction is deformed by adjoint couplings and consequently the constraints from the invisible decay become slightly tight. We see that a larger parameter region can be probed in a future, especially when the soft mass  $m_R$  is small.

As a reference, we show two sets of sample parameters in Table III. We take the averaged Higgsino mass  $\mu$  equal to  $M_S$ , while the ratio is given by  $\mu_u/\mu_d = \tan \varphi / \tan \beta = 1.25$ . The values of  $\tan \alpha$  correspond to our input that the pseudo-scalar Higgs mass is equal to  $M_S$ . The value of  $\tan \alpha$  follows from taking the pseudo-scalar Higgs mass equal to  $M_S$ , and implies 0.2% enhancement (0.8% reduction) of the up-type (down-type) Yukawa interactions. The adjoint scalar masses according to  $m_{\sigma_a}^2/8 = 8m_{\pi_a}^2 = m_{D_a}^2$  for each  $a = 1, 2, 3$  lead to the  $D$ -term suppression  $\varepsilon_D = 1/2$ .

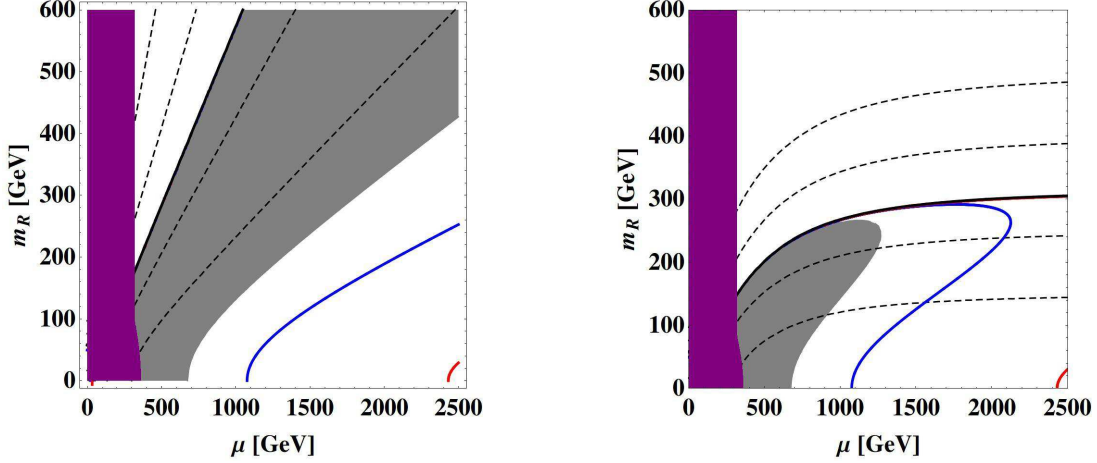


FIG. 10. The same as in Fig. 9, but with the adjoint Yukawa couplings  $\lambda_A = 0.4$ .

Input	$M_S (= \mu)$	$\tan \beta$	$M_{D_{1,2,3}}$ [TeV]	$m_{R_{0,u,d}}$ [GeV]	$\tan \varphi$	$\tan \alpha$
Case 1	1 TeV	2	(1.0, 3.0, 7.0)	(24, 120, 120)	2.5	-0.45
Case 2	2 TeV	2	(2.0, 3.0, 4.8)	(0, 260, 260)	2.5	-0.46
Output	$\Lambda$ [GeV]	$\lambda(M_S)$	$\varepsilon_D$	$\varepsilon_R$	$m_\phi$	$\text{Br}_{h \rightarrow \text{inv}}$
Case 1	$1.0 \times 10^{17}$	0.73	0.5	0.02	28 GeV	11%
Case 2	$4.0 \times 10^{16}$	0.69	0.5	0.02	15 GeV	13%

TABLE III. Sample sets of input/output parameters. The suppression factors  $\varepsilon_D$ ,  $\varepsilon_R$  and the mixing angle  $\tan \varphi$  are defined by Eqs. (8), (26) and (14), respectively.

## V. SUMMARY AND DISCUSSION

We have studied a singlet extension of the minimal  $R$ -symmetric SUSY SM with Dirac gauginos that is consistent with gauge coupling unification. Specifically we have calculated the mass of the SM-like Higgs boson and found that its observed value of 125 GeV can well be reproduced in a small  $\tan \beta$  region, by the nMSSM-like, singlet Yukawa coupling  $\lambda$  within perturbativity up to the unification scale  $10^{16}$ – $10^{17}$  GeV. This is true even when the SUSY scale is as low as 1 TeV and, remarkably, even without the standard  $D$ -term contribution to the quartic Higgs potential. The latter is important because the  $D$ -term potential is known to be suppressed in theories of Dirac gauginos and supersoft SUSY breaking.

The unification is preserved in a minimal way by adding two vector-like pairs of singlet “leptons”. Adding more extra matters makes the gauge couplings stronger at UV and thus relaxes the triviality bound. We also examined the precision of unification by using two-loop RGE’s. Although we have not taken into account the full variety of SUSY particle threshold, we found that a heavier Dirac gluino and/or wino makes the unification precise. We note that the mass threshold of a Dirac gaugino combined with its scalar partners,  $\sigma$  and  $\pi$ , can conveniently be represented at one-loop by a single mass scale  $M_D = (m_D^8 m_\sigma m_\pi)^{1/10}$ .

The allowed parameter space is rather limited in the small  $\tan \beta$  region especially when



$SU(2) \times U(1)$   $D$ -terms are suppressed by adjoint scalar contributions. Recent dedicated calculations [20, 28] show, however, that a part of two-loop contributions give substantial improvement to the Higgs mass in the singlet extensions of the MSSM; This is also the case in models that contains Dirac gluino. It would imply that our calculation based on RG method may underestimate the upper bound of the Higgs mass and that there is broader parameter region that is consistent with the triviality bound.

A characteristic feature of the present  $R$ -symmetric model is the existence of light scalar and fermion modes, pseudo-moduli  $\phi$  and pseudo-Goldstino  $\psi$ , whose properties are restricted by the approximate  $U(1)_R$  symmetry. Although the mixing between the pseudo-moduli and the SM Higgs is negligible, it does affect the RG evolution of the SM Higgs quartic coupling below the SUSY scale. We have examined this effect by constructing the SM coupled with the pseudo-moduli and found that the Higgs mass is reduced by a few %, but there exists a parameter region consistent with 125 GeV Higgs, again even without the  $D$ -term.

As we discussed, the constraints from invisible decays of the  $Z$  and Higgs bosons are weak if the singlet component dominates the lightest mass eigenstates. The constraint from the  $Z$  decay can easily be satisfied since the present model contains the Higgsino mass parameters. On the other hand, the invisible Higgs decay generically gives a tight constraint, as was originally expected. We have found, however, that there are interesting regions of parameter space in which the Higgs invisible width is within the current bound and within the proposed reaches of the future experiments: This occurs especially when the Higgsino mass parameters are much larger than  $\lambda v$  of the order of 100 GeV.

There remain many issues to be discussed. First, improved calculations including the full SUSY spectrum and higher-loop corrections are desired. The invisible Higgs decay in the  $R$ -symmetric setup should be studied in more general manner; we have explored a limited region of parameter space. For instance, if the singlet soft mass is larger than the doublet ones, the lightest mass eigenstate  $\phi$  deviates from the original pseudo-moduli direction. This implies that the  $\phi$  becomes heavier but has a larger coupling to the Higgs. Loop corrections may also be important since the suppression of the pseudo-moduli coupling to the Higgs boson depends on a specific structure of the potential. Detailed studies on the adjoint Yukawa couplings and their upper bounds should be done. On the theoretical side, it is also important to justify the assumptions about the origin or the absence of superpotential terms that we mentioned in §II. Finally we should remark that the cosmology of the singlet extended  $R$ -symmetric model is still challenging and deserves further study.

## ACKNOWLEDGMENT

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## Appendix A: Notes on RG Analysis

Here we summarize necessary tools for RG analysis: input parameters, one- and two-loop RGE's below and above the SUSY scale  $M_S$ . Two-loop RGE's are generated by the Mathematica package SARAH [58–60]. We write two-loop beta function for a generic coupling  $g_i$  in the form  $dg_i/dt = \beta_{g_i}^{(1)}/(16\pi^2) + \beta_{g_i}^{(2)}/(16\pi^2)^2$ , where  $t = \log(Q/m_Z)$  is a logarithm of the renormalization scale  $Q$ . We mainly use  $g_1^2 = (5/3)g_Y^2$  for  $U(1)$  coupling.

### 1. Input Parameters

The input parameters at the EW scale are [55]

$$\alpha_s(m_Z) = 0.1184 \text{ GeV} , \quad (\text{A1})$$

$$m_h = 125.7 \text{ GeV} , \quad (\text{A2})$$

$$m_t = 173.21 \pm 0.87 \text{ GeV} . \quad (\text{A3})$$

We use the center value for  $m_t$  because its error gives negligibly small effects in our analysis. The parameters at the top mass scale in the  $\overline{\text{MS}}$  scheme are given by [61]

$$g_s(m_t) = 1.1666 - 0.00046 \left( \frac{m_t}{\text{GeV}} - 173.10 \right) , \quad (\text{A4})$$

$$g_2(m_t) = 0.64822 + 0.00004 \left( \frac{m_t}{\text{GeV}} - 173.10 \right) \quad (\text{A5})$$

$$g_Y(m_t) = 0.35761 + 0.00011 \left( \frac{m_t}{\text{GeV}} - 173.10 \right) , \quad (\text{A6})$$

$$y_t(m_t) = 0.93558 + 0.00550 \left( \frac{m_t}{\text{GeV}} - 173.10 \right) , \quad (\text{A7})$$

$$\frac{1}{2}\lambda_H(m_t) = 0.12711 + 0.00206 \left( \frac{m_h}{\text{GeV}} - 125.66 \right) - 0.00004 \left( \frac{m_t}{\text{GeV}} - 173.10 \right) . \quad (\text{A8})$$

### 2. SUSY RGE's with Singlet and Adjoint Yukawa Couplings

Here we show the relevant RGE's in our  $R$ -symmetric SUSY model that include the singlet and adjoint Yukawa couplings,  $\lambda$  and  $\lambda_{ai}$ , defined by the superpotentials (1) and (3), respectively. We neglect the bottom and tau Yukawa couplings since eventually we are interested in a small  $\tan\beta$  region. Then one-loop beta functions for the top Yukawa coupling  $y_t$  and the singlet Yukawa coupling  $\lambda$  are given by

$$\beta_{y_t}^{(1)} = y_t \left( 6y_t^2 + \lambda^2 + |\lambda_S^u|^2 + 3|\lambda_T^u|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right) , \quad (\text{A9})$$

$$\beta_\lambda^{(1)} = \lambda \left( 3y_t^2 + 4\lambda^2 + |\lambda_S^u|^2 + |\lambda_S^d|^2 + 3|\lambda_T^u|^2 + 3|\lambda_T^d|^2 - 3g_2^2 - \frac{3}{5}g_1^2 \right), \quad (\text{A10})$$

and for the adjoint Yukawa couplings  $\lambda_{ai}$  ( $a = S, T$  and  $i = u, d$ ),

$$\beta_{\lambda_T^u}^{(1)} = \lambda_T^u \left( \lambda^2 + 2|\lambda_S^u|^2 + 2|\lambda_T^d|^2 + 8|\lambda_T^u|^2 - \frac{3}{5}g_1^2 - 7g_2^2 + 3y_t^2 \right), \quad (\text{A11})$$

$$\beta_{\lambda_T^d}^{(1)} = \lambda_T^d \left( \lambda^2 + 2|\lambda_S^d|^2 + 8|\lambda_T^d|^2 + 2|\lambda_T^u|^2 - \frac{3}{5}g_1^2 - 7g_2^2 \right), \quad (\text{A12})$$

$$\beta_{\lambda_S^u}^{(1)} = \lambda_S^u \left( \lambda^2 + 2|\lambda_S^d|^2 + 4|\lambda_S^u|^2 + 6|\lambda_T^u|^2 - \frac{3}{5}g_1^2 - 3g_2^2 + 3y_t^2 \right), \quad (\text{A13})$$

$$\beta_{\lambda_S^d}^{(1)} = \lambda_S^d \left( \lambda^2 + 4|\lambda_S^d|^2 + 2|\lambda_S^u|^2 + 6|\lambda_T^d|^2 - \frac{3}{5}g_1^2 - 3g_2^2 \right). \quad (\text{A14})$$

We write our two-loop beta functions in the form  $\beta_{g_i}^{(2)} = \beta_{g_i, \text{MDGSSM}}^{(2)} + \Delta\beta_{g_i}^{(2)}$ , where  $\beta_{g_i, \text{MDGSSM}}^{(2)}$  are beta functions in the minimal Dirac Gaugino SUSY SM (MDGSSM) [17] with  $U(1)_R$ -violating couplings there,  $\lambda_S$ ,  $\lambda_T$ ,  $\lambda_{SR}$ ,  $Y_{\hat{E}i}^{ij}$ ,  $Y_{\text{LFV}}^j$ ,  $Y_{\text{EFV}}^j$ ,  $\lambda_{ST}$ ,  $\lambda_{SO}$ ,  $\kappa_o$ , as well as extra lepton couplings  $\lambda_{S\hat{E}ij}$  and  $Y_{\hat{E}i}$ , all set to zero. The deviations  $\Delta\beta_{g_i}^{(2)}$  are found as follows: For gauge couplings  $g_a$  ( $a = 1, 2, 3$ ),

$$\Delta\beta_{g_1}^{(2)} = -\frac{6}{5}\lambda^2 g_1^3, \quad \Delta\beta_{g_2}^{(2)} = -2\lambda^2 g_2^3, \quad \Delta\beta_{g_3}^{(2)} = 0, \quad (\text{A15})$$

and for the MSSM Yukawa couplings,

$$\Delta\beta_{Y_u}^{(2)} = \lambda Y_u \left[ -3\lambda^2 - 3|\lambda_T^d|^2 - |\lambda_S^d|^2 - Y_d^\dagger Y_d - 3Y_u^\dagger Y_u - 3\text{Tr}(Y_d Y_d^\dagger) - \text{Tr}(Y_e Y_e^\dagger) \right], \quad (\text{A16})$$

$$\Delta\beta_{Y_d}^{(2)} = \lambda Y_d \left[ -3\lambda^2 - 3|\lambda_T^u|^2 - |\lambda_S^u|^2 - Y_u^\dagger Y_u - 3Y_d^\dagger Y_d - 3\text{Tr}(Y_u Y_u^\dagger) \right], \quad (\text{A17})$$

$$\Delta\beta_{Y_e}^{(2)} = \lambda Y_e \left[ -3\lambda^2 - 3|\lambda_T^u|^2 - |\lambda_S^u|^2 - 3Y_e^\dagger Y_e - 3\text{Tr}(Y_u Y_u^\dagger) \right]. \quad (\text{A18})$$

For the adjoint Yukawa couplings  $\lambda_{ai}$ , we have

$$\Delta\beta_{\lambda_T^u}^{(2)} = \lambda^2 \lambda_T^u \left[ -3\lambda^2 - 5|\lambda_T^d|^2 - 5|\lambda_T^u|^2 - |\lambda_S^d|^2 - |\lambda_S^u|^2 - 3\text{Tr}(Y_d Y_d^\dagger) - \text{Tr}(Y_e Y_e^\dagger) \right], \quad (\text{A19})$$

$$\Delta\beta_{\lambda_T^d}^{(2)} = \lambda^2 \lambda_T^d \left[ -3\lambda^2 - 5|\lambda_T^d|^2 - 5|\lambda_T^u|^2 - |\lambda_S^d|^2 - |\lambda_S^u|^2 - 3\text{Tr}(Y_u Y_u^\dagger) \right], \quad (\text{A20})$$

$$\Delta\beta_{\lambda_S^u}^{(2)} = \lambda^2 \lambda_S^u \left[ -3\lambda^2 - 3|\lambda_T^d|^2 - 3|\lambda_T^u|^2 - 3|\lambda_S^d|^2 - 3|\lambda_S^u|^2 - 3\text{Tr}(Y_d Y_d^\dagger) - \text{Tr}(Y_e Y_e^\dagger) \right], \quad (\text{A21})$$

$$\Delta\beta_{\lambda_S^d}^{(2)} = \lambda^2 \lambda_S^d \left[ -3\lambda^2 - 3|\lambda_T^d|^2 - 3|\lambda_T^u|^2 - 3|\lambda_S^d|^2 - 3|\lambda_S^u|^2 - 3\text{Tr}(Y_u Y_u^\dagger) \right]. \quad (\text{A22})$$

Finally and most importantly, for the singlet Yukawa coupling  $\lambda$ , we have

$$\beta_\lambda^{(2)} = \beta_\lambda^{(2,0)} + \beta_\lambda^{(2,1)} + \beta_\lambda^{(2,2)}, \quad (\text{A23})$$

$$\beta_\lambda^{(2,0)} = -10\lambda^5 + 2 \left( \frac{3}{5}g_1^2 + 3g_2^2 \right) \lambda^3 + \frac{3}{2} \left( \frac{99}{25}g_1^4 + \frac{6}{5}g_1^2 g_2^2 + 11g_2^4 \right) \lambda, \quad (\text{A24})$$

where  $\beta_\lambda^{(2,1)}$  and  $\beta_\lambda^{(2,2)}$  are contributions from the adjoint and the MSSM Yukawa couplings,

$$\beta_\lambda^{(2,1)} = -3\lambda^3 \left( 3|\lambda_T^u|^2 + 3|\lambda_T^d|^2 + |\lambda_S^u|^2 + |\lambda_S^d|^2 \right) - 3\lambda \left( 5|\lambda_T^u|^4 + 5|\lambda_T^d|^4 + |\lambda_S^u|^4 + |\lambda_S^d|^4 \right)$$

$$\begin{aligned}
& -4\lambda \left( 3|\lambda_T^u|^2 |\lambda_T^d|^2 + |\lambda_S^u|^2 |\lambda_S^d|^2 \right) - 6\lambda \left( |\lambda_S^u|^2 |\lambda_T^u|^2 + |\lambda_S^d|^2 |\lambda_T^d|^2 \right) \\
& + 12\lambda g_2^2 \left( |\lambda_T^u|^2 + |\lambda_T^d|^2 \right) ,
\end{aligned} \tag{A25}$$

$$\begin{aligned}
\beta_\lambda^{(2,2)} = & -3\lambda^3 \left[ 3\text{Tr}(Y_u Y_u^\dagger) + 3\text{Tr}(Y_d Y_d^\dagger) + \text{Tr}(Y_e Y_e^\dagger) \right] \\
& - 3\lambda \left[ 3\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) + 3\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) + 2\text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) + \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \right] \\
& + 16\lambda g_3^2 \left[ \text{Tr}(Y_u Y_u^\dagger) + \text{Tr}(Y_d Y_d^\dagger) \right] \\
& + \frac{2}{5}\lambda g_1^2 \left[ \text{Tr}(Y_d Y_d^\dagger) + 2\text{Tr}(Y_u Y_u^\dagger) + 3\text{Tr}(Y_e Y_e^\dagger) \right] .
\end{aligned} \tag{A26}$$

### 3. RGE's in the SM coupled with a complex scalar

Here we show two-loop RGE's in the SM coupled with a complex scalar  $\phi$ , whose quartic potential is given by Eq. (24). In our case, such theory is used in §III E as a low-energy effective theory of our  $R$ -symmetric model containing the light pseudo-moduli  $\phi$ .

For the effective theory couplings  $g_i = \lambda_H, \lambda_{\phi H}$  and  $\lambda_\phi$ , one-loop beta functions  $\beta_{g_i}^{(1)}$  are shown in Eqs. (27)–(31). For the SM couplings, we write two-loop beta functions in the form  $\beta_{g_i}^{(2)} = \beta_{g_i, \text{SM}}^{(2)} + \Delta\beta_{g_i}^{(2)}$ , where the SM contribution can be found for instance in Refs. [61, 62]. For the gauge couplings  $g_a$  ( $a = 3, 2, 1$ ), the Yukawa coupling matrices  $Y_f$  ( $f = u, d, e$ ) and the Higgs quartic coupling  $\lambda_H$ , the new contributions  $\Delta\beta_{g_i}^{(2)}$  are found to be

$$\Delta\beta_{g_a}^{(2)} = 0 , \quad \Delta\beta_{Y_f}^{(2)} = \frac{1}{2}Y_f\lambda_{\phi H}^2 , \quad \Delta\beta_{\lambda_H}^{(2)} = -10\lambda_H\lambda_{\phi H}^2 - 8\lambda_{\phi H}^3 . \tag{A27}$$

For the couplings  $\lambda_{\phi H}$  and  $\lambda_\phi$  in the effective theory (24), we have

$$\begin{aligned}
\beta_{\lambda_{\phi H}}^{(2)} = & \left( \frac{1671}{400}g_1^4 + \frac{9}{8}g_1^2g_2^2 - \frac{145}{16}g_2^4 \right) \lambda_{\phi H} + \left( \frac{3}{5}g_1^2 + 3g_2^2 \right) (12\lambda_H + \lambda_{\phi H}) \lambda_{\phi H} \\
& - \lambda_{\phi H} \left[ 15\lambda_H^2 + \frac{5}{2}\lambda_\phi^2 + 12(3\lambda_H + \lambda_\phi)\lambda_{\phi H} + 11\lambda_{\phi H}^2 \right] \\
& + \lambda_{\phi H} \text{Tr}(Y_u Y_u^\dagger) \left[ \frac{17}{4}g_1^2 + \frac{45}{4}g_2^2 + 40g_3^2 - 12(3\lambda_H + \lambda_{\phi H}) \right] \\
& + \lambda_{\phi H} \text{Tr}(Y_d Y_d^\dagger) \left[ \frac{5}{4}g_1^2 + \frac{45}{4}g_2^2 + 40g_3^2 - 12(3\lambda_H + \lambda_{\phi H}) \right] \\
& + \lambda_{\phi H} \text{Tr}(Y_e Y_e^\dagger) \left[ \frac{15}{4}g_1^2 + \frac{15}{4}g_2^2 - 4(3\lambda_H + \lambda_{\phi H}) \right] - 21\lambda_{\phi H} \text{Tr}(Y_d Y_u^\dagger Y_u Y_d^\dagger) \\
& - \frac{9}{2}\lambda_{\phi H} \left[ 3\text{Tr}(Y_u Y_u^\dagger Y_u Y_u^\dagger) + 3\text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) + \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) \right] ,
\end{aligned} \tag{A28}$$

$$\begin{aligned}
\beta_{\lambda_\phi}^{(2)} = & - \left( 15\lambda_\phi^3 + 20\lambda_\phi\lambda_{\phi H}^2 + 32\lambda_{\phi H}^3 \right) + 16\lambda_{\phi H}^2 \left( \frac{3}{5}g_1^2 + 3g_2^2 \right) \\
& - 16\lambda_{\phi H}^2 \left[ 3\text{Tr}(Y_u Y_u^\dagger) + 3\text{Tr}(Y_d Y_d^\dagger) + \text{Tr}(Y_e Y_e^\dagger) \right] .
\end{aligned} \tag{A29}$$

## Appendix B: A Note on Dirac Gaugino Mass Threshold

Let us consider an  $SU(N)$  gauge theory that contains a gaugino  $\lambda_a$ , its Dirac partner  $\chi_a$ , and their scalar partners,  $A_a = (\sigma_a + i\pi_a)/\sqrt{2}$ , all in the adjoint representations. We are interested in the mass threshold in the one-loop running of the gauge coupling constant. We will show that the mass threshold can be represented by a single mass threshold  $M_D$  defined by

$$M_D = [m_D^8 m_\sigma m_\pi]^{1/10} . \quad (\text{B1})$$

To see this, let us look at the solution to one-loop gauge RGE in the form

$$\frac{2\pi}{\alpha(Q)} - \frac{2\pi}{\alpha(Q_0)} = b_0 \ln \frac{Q_0}{Q} + \sum_i \Delta_i b \ln \frac{m_i}{Q} , \quad (\text{B2})$$

where  $b_0$  is the massless contribution to the beta function coefficient, and  $\Delta_i b$  is a contribution of the  $i$ -th particle of mass  $m_i$ . The sum is taken over all the particles whose mass lies between the renormalization scale  $Q$  and the reference scale  $Q_0$  at IR. For the case of interest, a Dirac pair of gaugino  $\lambda_\alpha$  and  $\chi_\alpha$  contributes  $\Delta_D b = 4N/3$  while its scalar partners give  $\Delta_\sigma b = \Delta_\pi b = N/6$ . If these fields were degenerate in mass,  $m_\sigma = m_\pi = m_D$ , we would have

$$\sum_{i=\lambda, \chi, \sigma, \pi} \Delta_i b \ln \frac{m_i}{Q} = \left( \frac{4N}{3} + \frac{2N}{6} \right) \ln \frac{m_D}{Q} = \frac{5N}{3} \ln \frac{m_D}{Q} . \quad (\text{B3})$$

Actually we have

$$\sum_{i=\lambda, \chi, \sigma, \pi} \Delta_i b \ln \frac{m_i}{Q} = \frac{4N}{3} \ln \frac{m_D}{Q} + \frac{N}{6} \ln \frac{m_\sigma}{Q} + \frac{N}{6} \ln \frac{m_\pi}{Q} = \frac{5N}{3} \ln \frac{M_D}{Q} . \quad (\text{B4})$$

We see that at the leading log level, the mass threshold effect due to the massive adjoint fields in a Dirac gaugino model can be represented by a single mass threshold scale  $M_D$  defined by Eq. (B1).

If we denote the holomorphic and non-holomorphic contributions to the adjoint scalar mass term by  $b$  and  $m_A^2$ , respectively, the squared masses of the real and imaginary components of the adjoint scalars are

$$\begin{aligned} m_\sigma^2 &\equiv \mathcal{Z}_\sigma m_D^2 = 4m_D^2 + b + m_A^2 , \\ m_\pi^2 &\equiv \mathcal{Z}_\pi m_D^2 = -b + m_A^2 . \end{aligned} \quad (\text{B5})$$

In principle these adjoint scalar masses can take any values, (although it involves a fine tuning). Therefore we can regard the coefficients  $\mathcal{Z}_{\sigma, \pi}$  as (positive) free parameters. Using the above parametrization, we have a relation  $M_D = (\mathcal{Z}_\sigma \mathcal{Z}_\pi)^{1/10} m_D$ .

The Dirac mass threshold  $M_D$  defined here coincides with the actual Dirac mass of the gaugino if  $\mathcal{Z}_\sigma \mathcal{Z}_\pi = 1$ , that is,

$$m_\sigma^2 = \mathcal{Z}_\sigma m_D^2 , \quad m_\pi^2 = \frac{1}{\mathcal{Z}_\sigma} m_D^2 , \quad (\text{B6})$$

Otherwise, the mass threshold scale does not coincide with the Dirac mass parameter itself. Notice that requiring that  $m_D = m_\sigma = m_\pi$  would imply a negative mass correction to the real scalar  $\sigma$ ,  $\delta m_\sigma^2 = m_A^2 + b = -3m_D^2$ .

### Appendix C: Examples of Gauge Coupling Unification

Here we give several examples in which gauge coupling unification is satisfied under our simplifying assumption stated in §III A. We use two-loop RGE's summarized in App. A and treat the SUSY threshold by a single representative scale  $M_S$  with the exception of the extra vector-like leptons and  $SU(3) \times SU(2)$  Dirac gauginos. The results are summarized in Table IV.

In the first example, the unification is achieved by tuning the bachelor mass  $M_E$ . Recall that two-loop contributions make the  $SU(3)$  gauge coupling slightly asymptotically non-free. This implies that the unification scale  $\Lambda$  becomes slightly larger than the one-loop value  $2 \times 10^{16}$  GeV. With the extra leptons heavier,  $U(1)_Y$  gauge coupling becomes smaller so that it can cross the intersecting point of  $SU(3) \times SU(2)$  couplings. This corresponds to parameter set (I) in Table IV and is depicted in Fig. 11.

More natural examples are provided by changing the Dirac mass thresholds,  $M_{D_3}$  and  $M_{D_2}$ , for the  $SU(3) \times SU(2)$  gauginos. Although  $M_{D_a}$  is not the same as the gaugino mass  $m_{D_a}$ , it is plausible that Dirac gluino is the heaviest and Dirac Wino is the next heaviest gaugino, due to an RG effect. A heavier Dirac gluino mass threshold implies a smaller  $SU(3)$  coupling at UV. By tuning the extra lepton mass, unification can be achieved at relatively lower energy scale around  $10^{16}$  GeV. In this way we obtain parameter sets (II) and (III) in Table IV. Two-loop running gauge couplings in parameter set (II) are shown in Fig. 12.

In passing, it is interesting to note that under our simplifying assumption, the extra leptons must be lighter than  $M_S = 1$  TeV if  $M_{D_3}$  is larger than 3 TeV; similarly, the extra leptons must be lighter than  $M_S = 2$  TeV if  $M_{D_3} > 6$  TeV.

Finally we consider the case in which the  $SU(2)$  gaugino mass threshold can also be different: parameter sets (IV), (V) and (VI) in Table IV. In this case it is not so easy to discuss the effect qualitatively because changing  $M_{D_2}$  affects the running of  $SU(3)$  coupling.

Case	$M_S$	$M_E$ [TeV]	$M_{D_{1,2,3}}$ [TeV]	$\Lambda$ [GeV]
(I)	1 TeV	10–15	(1.0, 1.0, 1.0)	$1.0 \times 10^{17}$
(II)		1	(1.0, 1.0, 3.0)	$3.0 \times 10^{16}$
(III)	2 TeV	2	(1.0, 1.0, 6.0)	$2.0 \times 10^{16}$
(IV)		6	(2.0, 3.0, 4.8)	$4.0 \times 10^{16}$
(V)	1 TeV	6	(1.0, 1.5, 2.4)	$1.0 \times 10^{17}$
(VI)		5	(1.0, 3.0, 7.0)	$5.0 \times 10^{16}$

TABLE IV. Sample sets of parameters for gauge coupling unification;  $\varepsilon_D = 0$ ,  $\tan \beta = 2$ .

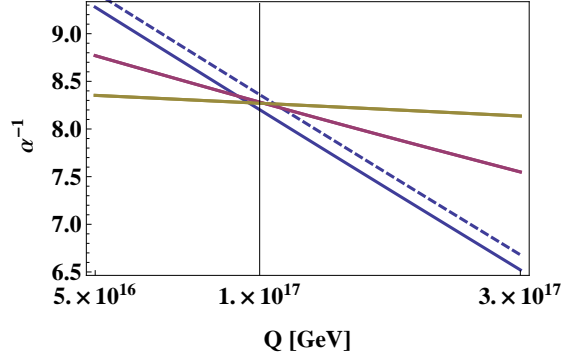
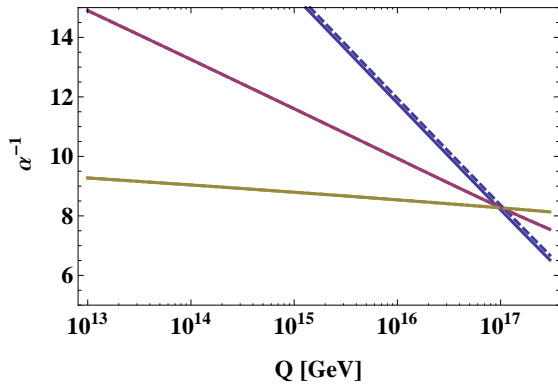


FIG. 11. Two-loop running of gauge couplings corresponding to parameter set (I) in Table IV. The extra lepton mass is  $M_E = 10$  TeV (solid) and 15 TeV (dashed).

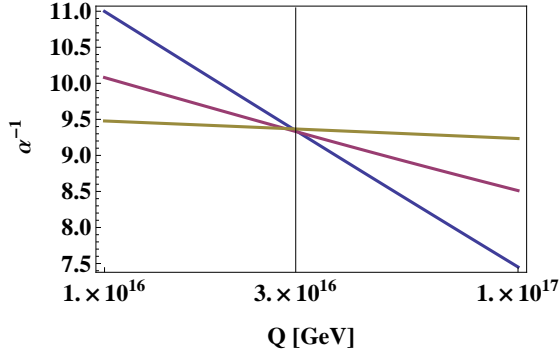


FIG. 12. Two-loop gauge running corresponding to parameter set (II):  $M_{D_3} = 3$  TeV and  $M_{D_2} = M_S = 1$  TeV.

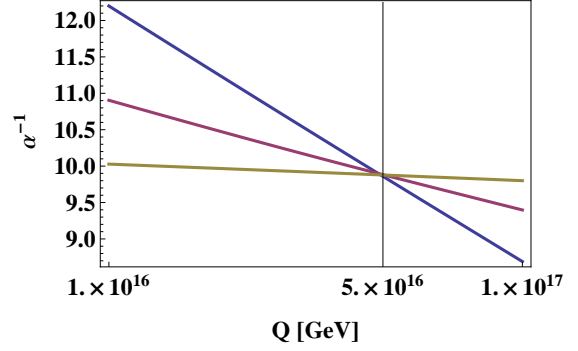


FIG. 13. Two-loop gauge running corresponding to parameter set (VI):  $M_{D_3} : M_{D_2} : M_{D_1} = 7 : 3 : 1$  TeV.

Parameter set (IV) is shown in Fig. 13.

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